2011 CAP Prize Examination

Compiled by
The Department of Physics and Astronomy, University of Calgary

Tuesday, February 8, 2011 Duration : 3 hours.

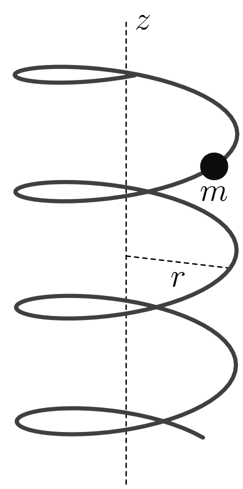
Instructions :

1. You are permitted to use calculators for the exam.
2. There are 10 questions on 12 pages. Page 12 is a list of constants.
3. Each question will be marked by a different examiner. **The answer to each question should be written on a separate page.** If more than one page is required for any question, then those pages should be stapled together separately from other questions.
4. The number of the question, the name of the candidate, and the name of the candidate's university and department should be clearly indicated on the first page of each answer.
5. Each question has equal value. You are not expected to attempt all of the questions! Relax, and attempt the questions on material that you are most familiar with or those questions that just look most interesting to you.
6. The completed examination papers should be sent by the Department Chairpersons to :
   Dr. William J.F. Wilson
   Department of Physics and Astronomy
   University of Calgary
   2500 University Drive NW
   Calgary, AB T2N 1N4

Question 1 : Mechanics

Here are two mechanics problems. Try both.

(a) A bead of mass m slides down a wire helix of radius r and pitch angle θ at a constant speed, v. The wire has a circular cross-section, and the long axis of the helix is vertical. Ignore air resistance. What is the coefficient of kinetic friction, μ_k, between the bead and the wire? Express your answer in terms of symbols given in this problem and any required constants.



(b) A loop ribbon of circumference c, width w and mass m, is spun with an angular velocity ω about its axis of symmetry. What is the tension in the ribbon?

Question 2 : Mechanics

Anyone who has stirred sugar or milk into their tea or coffee has noticed that when the beverage is spinning around in the cup it forms a depression in the centre.

(a) Explain why the surface of a spinning liquid has this shape.

(b) Suppose that you placed your cup filled with stationary liquid in the center of a stationary turntable, and slowly increased its angular velocity. Will the surface of the liquid attain the same shape as in (a) after a while ? What if the interface between the liquid and the cup was perfectly frictionless ?

(c) Now you install a video camera directly above the turntable, aligned with the axis of rotation, so that the liquid surface is displayed on the monitor. The camera always rotates at exactly the same angular velocity as the turntable, so that on the screen the cup always appears to be at rest. Consider again the scenarios in part (b). What has changed, or remains the same ? Are the lab and co-rotating frames completely equivalent ? Why ?

(d) Suppose that the coffee was so exceptionally strong that it became as viscous as corn syrup. If I add a few drops of milk and then rotate the cup on the turntable a few revolutions, will the milk mix with the coffee ? If I then reverse the turntable exactly the same number of revolutions, will the milk unmix and reform a series of drops ? Suppose I had decided to add honey instead of milk (with approximately the same viscosity as the coffee) ; does anything change ? Why ?

(e) Suppose you always take two tablespoons of milk with each cup of coffee. After pouring the boiling liquid into your cup, you always wait exactly three minutes before you take a sip. If you want the drink to be hottest at this time, is it better to add the milk immediately after pouring the coffee, just before you take a sip, or sometime in between ? Why ?

Question 3 : Oscillations and Waves

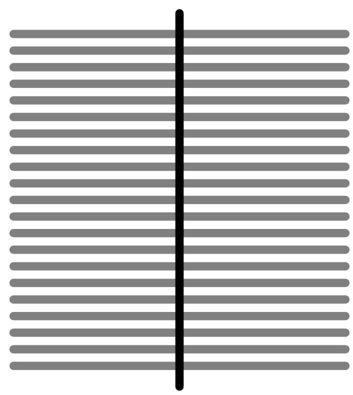
On a double bass the A-string is 2.00 metres in length and has a mass of 0.0100 kg. It vibrates in its fundamental mode under a tension of 3943 N. When the concert master plays the A-string in her violin (tuned exactly to 440 Hz.), the double bass player can hear beats between the sound of the violin and one of the harmonics of his A-string. He wishes to match the frequencies exactly by changing the tension in the string and placing his finger on the finger board to shorten the effective length of the string.

(a) What is the fundamental frequency and the first two harmonics of the A-string of the double bass before adjustment ?

(b) Explain exactly what needs to be done and why in order for the A-string of the double bass to match the frequency of 440 Hz played on the violin.

Question 4 : Oscillations and Waves

A wave demonstrator consists of an array of closely-spaced, horizontal rods, all parallel to each other a distance Δx apart (centre-to-centre), and connected together by a torsional spring that runs along the centre of array, perpendicular to the rods (see figure below). The torsional spring constant is κ, and each rod is of length l and of mass m. The first rod is twisted some small amount ξ_o, creating a torsional wave which propagates without loss along the wave demonstrator.



View from above

(a) Derive the wave equation for this situation.

(b) What is the wave speed, c_w?

(c) If the displacement of the wave obeys the equation $\xi = \xi_o \sin(kx - \omega t)$, how much power is transmitted with this wave as a function of m, l, ω, c_w, Δx and ξ_o?

Question 5 : Thermodynamics and statistical mechanics

If liquid quartz is cooled slowly, it crystallizes at a temperature T_m, and releases latent heat. If it cools more rapidly, the liquid is supercooled and becomes glassy.

(a) The liquid and solid phases of quartz are almost incompressible, so there is no work input and changes in internal energy satisfy $\mathrm{d}E = T\mathrm{d}S + \mu\mathrm{d}N$. Use the extensivity condition to obtain the expression for μ in terms of E, T, S and N.

(b) The heat capacity of crystalline quartz is approximately $C_x = \alpha T^3$, while that of glassy quartz is roughly $C_G = \beta T$, where α and β are constants. Assuming that the third law of thermodynamics applies to both crystalline and glass phases, calculate the entropies of the two phases at temperatures $T \leq T_m$.

(c) At zero temperature the local bonding structure is similar in glass and crystalline quartz, so that they have approximately the same internal energy, E_0. Calculate the internal energies of both phases at temeratures $T \leq T_m$.

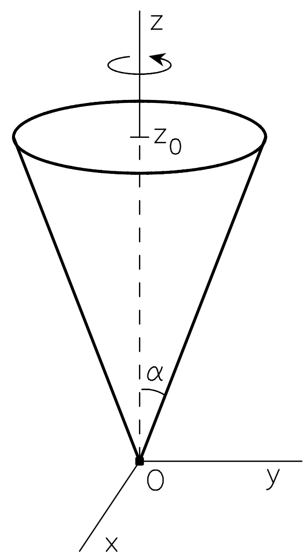
(d) Use the condition of thermal equilibrium between two phases to compute the equilibrium melting temperature, T_m, in terms of α and β.

(e) Compute the latent heat, L, in terms of α and β.

(f) Is the result in the previous part correct ? If not, which of the steps leading to it is most likely to be incorrect ?

Question 6 : Electromagnetism

The figure below shows a non-conducting cone with uniform surface charge density $\sigma = 10.6$ C/m^2. The axis of the cone is along the z-axis, and the point of the cone is at the origin, O. The half-opening angle of the cone is $\alpha = 30°$ (see the figure below), and the height of the cone along the z-axis is $z_0 = 20$ cm. The cone is spinning around the z-axis with angular frequency $\omega = 60.0$ Hz in the direction indicated in the figure.



(a) Show that at height z ($0 \leq z \leq 20$ cm), an infinitesimal part of the surface of the cone between z and $z + \mathrm{d}z$ can be considered a circular current loop with radius $r = z \tan\alpha$ carrying a current with magnitude

$$i = \frac{\omega r \sigma \mathrm{d}z}{\cos\alpha}$$

(b) Calculate the magnitude and the direction of the magnetic field at the origin, O. *Hint : Use the relation* $(\tan^2\alpha + 1) = 1/(\cos^2\alpha)$.

(c) At time $t = 0$ an increasing uniform magnetic field $\mathbf{B} = 3.00 \times 10^{-4} t\,\hat{\mathbf{z}}$ T is switched on. Calculate the rate of change of the rotational energy of the spinning cone. Also explain whether ω increases or decreases. *Hint : You do not need to know the actual rotational energy of the spinning cone.*

Question 7 : Physics Applications

A proton is moving in a region where there is a uniform but time changing magnetic field :

$$\mathbf{B} = B_\circ(1 + \alpha t)\,\hat{\mathbf{z}}$$

where $B_\circ$ and α are constants). The velocity of the proton has zero z-component (e.g., the motion is orthogonal to the magnetic field). The velocity is such that the problem is non-relativistic, and α is small enough that you can assume that any given cycle of the proton motion is a perfect circle. Use B and K to represent the magnetic field strength and proton kinetic energy, respectively.

(a) What is the change of kinetic energy of the proton in one cycle of its motion?

(b) Show that if the proton kinetic energy and magnetic field strength changes are ΔK and ΔB, then $B\Delta K = K\Delta B$.

(c) Show that K/B is constant under these conditions (this quantity is referred to as the *First Adiabatic Invariant* in Plasma Physics).

Question 8 : Optics

Consider light with a centre wavelength $\lambda = 633$ nm. Assume its power-spectrum is Gaussian and characterized by a Half Width at $1/\sqrt{e}$ Max of $\sigma_\nu = 1$ GHz. Assume the phase spectrum to be zero for all frequencies.

(a) Derive an expression for the intensity at the output of a Michelson Interferometer as a function of $\delta = 2\Delta x$, where Δx denotes the difference of the distance of the two mirrors from the semi-transparent mirror. Sketch the expression (not to scale).

(b) What should be the path length difference δ so that the Visibility drops to one half of its maximum value?

Question 9 : Quantum mechanics : Schrödinger's cat

(a) Why do we not usually use quantum physics in our daily lives ? For example, consider a moving cat. Estimate how well we know its position and momentum in practice. Compare the product of your estimated uncertainties in position and momentum for the cat with the Heisenberg uncertainty relation.

(b) Now consider a radioactive nucleus that can emit an alpha particle in a random direction. Suppose that the kinetic energy of the alpha particle is 1 MeV. Estimate its momentum uncertainty due to the fact that the direction of emission is uncertain.

(c) Suppose that if the alpha particle is emitted in a certain direction, it will trigger a mechanism that kills the cat. If it is emitted in other directions, it will not. We thus don't know if the cat is still moving or not. Explain how the original small momentum uncertainty for the alpha particle now leads to a large momentum uncertainty for the cat.

(d) Can you think of other examples where small quantum uncertainties are amplified to the macroscopic level ?

Question 10 : Physics Applications

Consider the production of a new particle Φ created from the head-on relativistic collision of two identical particles $p_1 = p_2 = p$ by the interaction :

$$p + p \rightarrow \Phi$$

It is known that the rest mass of the Φ particle is eight times that of the rest mass of the p particle.

(a) What is the minimum relativistic kinetic energy of each p-particle required to produce a Φ if the collision occurs between the two particles moving with the same speed but in opposite directions ?

(b) If the collision occurs such that one of the p's is initially at rest, what is the minimum kinetic energy of the incident particle required to produce a Φ particle from the collision ?

(c) What is the final velocity of the Φ particle in both cases ?

Constants used in this examination :

$c = 3.00 \times 10^{8}\ m\,s^{-1}$

$e = 1.60 \times 10^{-19}\ C$

$\hbar = 1.05 \times 10^{-34}\ J\,s$

$\mu_0 = 4\pi \times 10^{-7}\ Wb\,A^{-1}\,m^{-1}$

$1\ \mathrm{u} = 1.66 \times 10^{-27}\ kg$