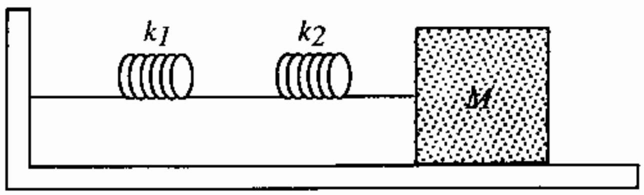
2004 Canadian Association of Physicists Prize Examination

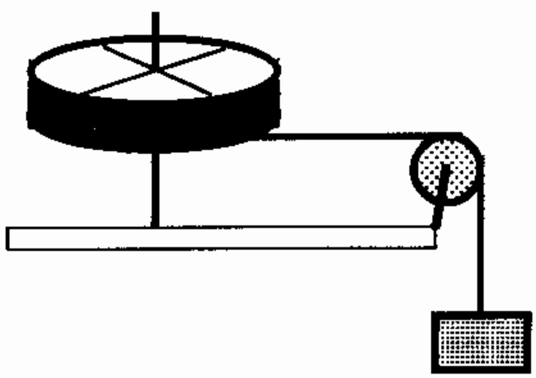
Tuesday, January 27, 2004

Duration: 3 hours

1. a. The mass M is connected to two springs with spring constants of k_1 and k_2. Friction between the masses and the table is negligible. Find the period T of the oscillating mass.



b. A cord is wrapped around a thin hoop, then passes over a pulley (made from a solid disk). Both the pulley and the hoop can rotate without friction about their central axes. A block is suspended from the end of the cord dangling down from the pulley. If the masses of the cylinder, hoop, and block are equal, what is the acceleration of the block, and the tension in all parts of the cord? Assume that the masses of the cord and spokes on the hoop are negligible, and that the cord does not slip when in contact with either the pulley or the hoop.



2. Find the nearest separation at which we can still see distinct objects positioned at the near-point of the normal eye (25.0 mm). The minimum diameter of the pupil is 2.00 mm, the diameter of the eye (an image distance) l = 25.0 mm, the refractive index of the eye is n_{eye}=1.33, the refractive index of air is n_{air} = 1.00. The eye is most sensitive to light of a wavelength of 555 nm (measured in air). You may find it helpful to know that the radius q of the first dark ring on an Airy disk can be given by $q = 1.22\frac{D\lambda}{2a}$, where D is the distance from an aperature of radius a to the plane of the disk, and λ is the wavelength of light used.

3. Challenger's Deep is the deepest part of the Marianas trench, in the Pacific Ocean off the Philippines. It is the deepest spot in any ocean on the planet at 10 924 m (nearly 11 km). The temperature at the bottom is about 3 C. A bubble of NO gas (which you may take to be an ideal gas), 1 μm in diameter, rises from the bottom of the Marianas trench to the surface of the Ocean.
a. If it manages to do so without bursting, what will be its diameter just below the surface, where the temperature is 35 C?
b. The expansion of the bubble is not a reversible process. Calculate the entropy change of the gas in the bubble.
You are given:
R = 8.3143 J/(K mol)
density of sea water   1000 kg/m3 and constant with temperature
g = 9.81 m/s2
c_V = 3R/2 J/K mol for an ideal monatomic gas
c_V = 5R/2 J/K mol for an ideal permanent diatomic gas
1 atm = 1.01 × 105 Pa = 760 mm Hg or Torr = 1.01 bar

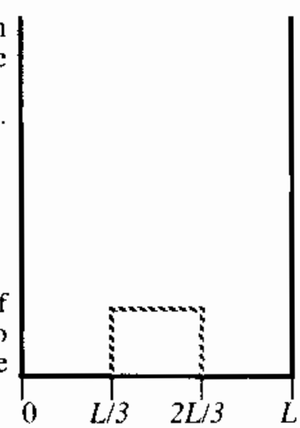
4. a. A particle of mass m_0 moving with a speed $0.8c$ makes a completely inelastic collision with a stationary particle of mass $3\,m_0$. What is the rest mass of the resulting body?

b. It has been proposed to use radiation as a spaceship propellant (a photon rocket), since the thrust is proportional to the speed of the ejected fuel. If the initial and final masses of the rocket are M_1 and M_2 show that the final velocity v of the rocket relative to its initial rest frame is given by $\frac{M_1}{M_2} = \sqrt{\frac{c+v}{c-v}}$, where c is the speed of light.

5. Consider a one-dimensional infinite square well potential spanning the length from 0 to L. The energy eigenvalues for a particle of mass m trapped in the well are $E_n = \frac{\pi^2 n^2 \hbar^2}{2mL^2}$, where n takes on integer values from 1 to infinity. The wavefunctions for the particle are proportional to $\sin\left(\frac{n\pi x}{L}\right)$.

a. What are the normalization factors for the each of the wavefunctions?

b. Suppose that the square well is perturbed by a small step function of height a that extends from $L/3$ to $2L/3$. Use first order perturbation theory to determine the change of the energy interval E_2-E_1 resulting from the presence of the step function.



c. Make a qualitative sketch the ground state wavefunction of the perturbed system. Give a brief written explanation of your answer.

d. Using first-order perturbation theory, the wavefunction you have sketched can be written in the form $|\Psi_1\rangle = |\psi_1\rangle + \sum_{p=2}^{\infty} a_p |\psi_p\rangle$, where $|\Psi_1\rangle$ is the ground state wavefunction of the perturbed system and the $|\psi_p\rangle$ are the wavefunctions of the unperturbed system. Which term do you expect to be dominant in the summation? Which coefficients a_p are identically zero? Briefly explain your answers.

6. A long cylinder of length L and radius R carries a uniform surface charge density σ. The cylinder (and hence the charge spins and accelerates about its axis (the z axis) with angular velocity $\vec{\omega} = \alpha_0 t\hat{z}$, where t is time.

   a. Find the magnetic field $\vec{B}$ in the x-y plane as a function of $\vec{r}$, the radial distance from the axis, and the time t (use Ampère's Law).

   b. Find the magnetic vector potential $\vec{A}$ in the x-y plane (hint: first find the magnetic flux).

   c. Find the induced electric field $\vec{E}_i$.

   You may neglect end effects in this problem ($r << L$, $R << L$).

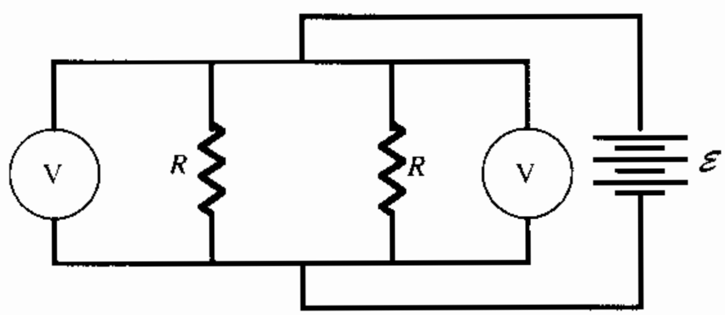
7. Consider a particle with spin $S = 1/2$.

   a. Find the eigenvalues and eigenfunctions of the operator $S_x + S_y$ where S_i is the spin operator in the i-direction ($i = x, y, z$).

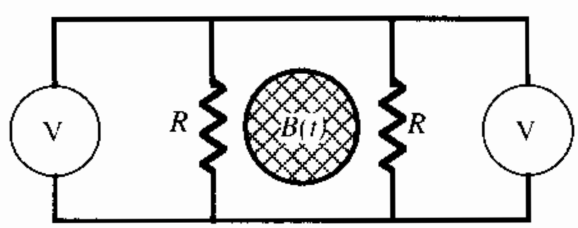
   b. Assume that $|\alpha\rangle$ designates the eigenfunction of $S_x + S_y$ that belongs to the maximal eigenvalue, and that the particle is in state $|\alpha\rangle$. If we measure the spin in the z-direction, what are the possible values we obtain and their probabilities?

   c. The particle is in the state $|\alpha\rangle$ described in part b. Find, if possible, the direction $\hat{n}$ in which the spin measurement will with certainty yield the value $S_n = +\hbar/2$.

8. a. Consider the circuit below. What is the reading on the voltmeters?



   b. Now the battery is removed from the circuit, and the circuit is subjected to a magnetic field $B(t)$ changing in time, in the region shown below. What now do the voltmeters read, in tems of the variables shown on the diagram?



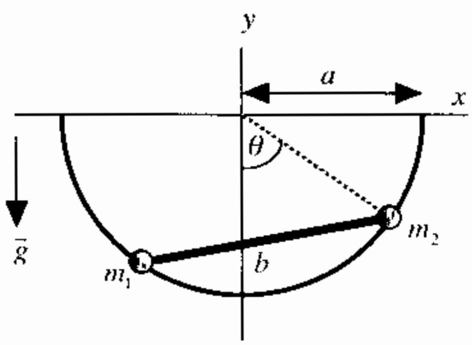
   c. Provide a qualitative explanation for your response to part b.

9. Two particles of masses m_1 and m_2 are constrained to move on a vertical circle of radius a as shown in the figure. They are connected by a light rod of length $b<2a$.

a. Find the Lagrangian in terms of θ and $\frac{d\theta}{dt}$.

b. Find an expression that would allow you to find θ_0, the angle when the system is in equilibrium.

c. Consider small-amplitude oscillations about θ_0 for the special case $m_1=m_2$. Show that the frequency of these oscillations is $\omega=\sqrt{\frac{g}{a}\cos\theta_0}$, where g is the acceleration due to gravity.



10. A pulsar emits a pulse of broadband electromagnetic radiation which is of 1 millisecond in duration. This pulse then propagates 1000 light years (10^{21} cm) through interstellar space to reach radio astronomers on Earth.

a. What must be the minimum bandwidth of a radio telescope receiver in order that the observed pulse shape is not to be distorted greatly?

b. Now consider that the interstellar medium contains a low density plasma (plasma frequency ω_p =5000 radians/sec). Estimate the difference in measured pulse arrival times for radio telescopes operating at 400 MHz and 1000 MHz, respectively. Recall that the dispersion relation for a plasma is $\omega^2 = k^2c^2 + \omega_p^2$.