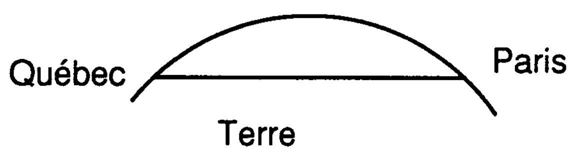
CAP University Prize Examination
Thursday, February 8, 2001, 14 :00-17 :00

Introduction

1. Calculators are allowed
2. Answer each question in a different booklet, with the question number, your name and that of your university on the first page.
3. The value attributed to each question takes into account the length and difficulty of the problem.
4. You are not expected to answer all questions. Please try as many as you can.

I(10)

We design a long range transport system by digging straigh line tunnels between surface destinations. Gravitational attraction provides all the needed acceleration and deceleration. High vacuum and magnetic levitation guarantee that no friction acts on the trains in the tunnels.



How long would it take to travel from Québec to Paris, from Québec to Toronto?

II(10)

Imagine a thermodynamic cycle followed in the 123 direction, where 12 is an adiabatic process, 23 an isothermic one and 31 an isochore decompression. This cycle represents the transformations of one kmole of a real gas in a machine (it remains gaseous).

1. Draw this cycle in the p-V plane
2. If the van der Walls equation is a good representation of this gas, what is the efficiency of this machine
3. Which of the two real gas effects affects most this efficiency, the finite volume of the molecules or the interaction between the molecules?

III(10)

It is possible to view a dielectric medium as made up of atoms on a lattice. Each atom has one electron of charge e bound to the site by a potential which we approximate by an harmonic potential of frequency ω in a Hamiltonian operator H_0. There are N atoms per unit volume. The dielectric medium is placed in an electric field $E\hat{z}$ (in the Oz direction). The field induces a polarisation (dipole) $\vec{P} = \varepsilon_0 \chi_e \vec{E}$ in the medium, where χ_e is the electric susceptibility of the medium.

1. Give an expression for χ_e as a function of the parameters of the problem.

2. Does the susceptibility χ_e increase or decrease when the frequency ω of the harmonic potential increase and why is it so .

IV(10)

A system contains N identical particles in a one dimensional space. They undergo interactions, which global result can be represented by an harmonic potential $V = \frac{m\omega^2 x^2}{2}$.

If the particles are in a quantum regime, they can occupy the states of energy $\varepsilon_n = (n + 1/2)\hbar\omega,\ n = 0,1,2,3...$

If the particles are in a classical regime, their energy is given by

$$\varepsilon = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}$$

1. Find the (classical) partition function for each regime.

2. Write an expression for the specific heat in each regime?

3. Do these specific heats differ at low temperature, at high temperature?

V(10)

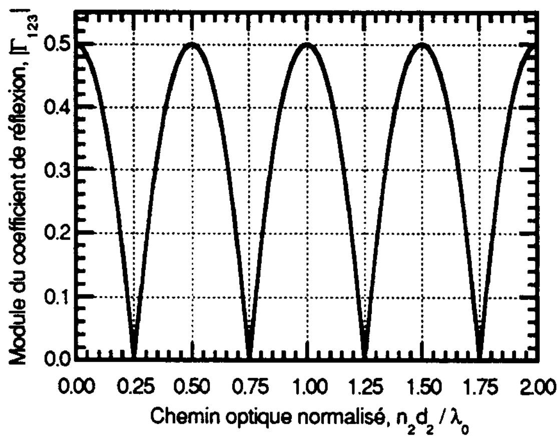
An observer at rest with respect to the fixed distant stars sees an isotropic distribution of stars. That is, in any solid angle $d\Omega$ he sees $dN = N(d\Omega/4\pi)$ stars, where N is the total number of stars he can see.

Suppose that another observer is moving at a relativistic velocity β in the $\hat{x}$ direction (rest frame system S'). What is the distribution of stars seen by this observer? Specifically, what is the distribution function $P(\theta',\phi')$ such that the number of stars seen by this observer in his solid angle $d\Omega'$ is $P(\theta',\phi')\,d\Omega'$? Check to see that $\int_{sphere} P(\theta',\phi')d\Omega' = N$ and that $P(\theta',\phi') \to \frac{N}{4\pi}$ as $\beta \to 0$.

Where will the observer see the stars *bunch up*?

VI(10)

You are asked to interpret the results obtained as a thin layer of material of index n_2 (n_2>1) is deposited on a very thick substrate (semi infinite in fact) of index n_3 (n_3>1). The surrounding medium is a vacuum (n_1=1). While depositing our layer (medium 2), the modulus of the reflection coefficient, $|\Gamma|$, (ratio of the amplitude of reflected wave over the amplitude of the incoming wave) was monitored as a function of the thickness of the layer. The result is reported on the figure where the vertical axis should read "Modulus of reflection coefficient $|\Gamma|$" and the horizontal should read "Normalised optical path $n_2 d_2 / \lambda_2$".



a. Using the graph, determine first the index n_3 of the substrate and then the index n_2 of the material in the thin layer.

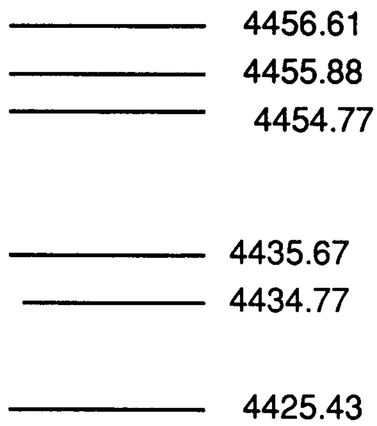
b. If the thickness of medium 2 equals $d_2 = \lambda_2 / 8$, what is the ratio of the (outside) stationary wave. Use the graphics to avoid a long calculation.

c. What would be the sign of Γ for $d_2 = \lambda_2 / 2$? Why?

VII(8)

The following figure shows the observed lines, with λ in angströms, in the spectrum of a certain atom of intermediary *Z*. These lines correspond to all the possible (optically allowed) transitions between the levels of two multiplets.

Determine the quantum numbers *LSJ* characterizing these multiplets and their levels. Explain your results.



VIII(7)

This problem finds some applications in biophysics where it can be used to model some neurophysical processes. A spherical current source is embedded in a medium where the conductivity behaves as $1/r^2$ with respect to the center of the source. Determine the electrical potential and the charge density everywhere outside the source.

IX(10)

1. Write down the law of radioactive decay. Define the half-life and mean life of a radioactive nucleus and obtain the relation between them.

2. The nucleus ^{87}Rb (Z=37) decays into the ground state of ^{87}Sr (Z=38), with a half-life of 4.7×10^{10} years and a maximum energy for the β of 272 keV. Discuss briefly the difficulties you might encounter in attempting to measure this half-life.

3. Five samples of chondritic meteorites are found to have the following proportions of ^{87}Rb, ^{87}Sr et ^{86}Sr.

Meteorites	^{87}Rb/^{86}Sr	^{87}Sr/^{86}Sr
Modoc	0.86	0.757
Homestead	0.8	0.751
Bruderheim	0.72	0.747
Kyushu	0.6	0.739
Bath Furnace	0.09	0.706

Given that the nucleus ^{86}Sr is not a daughter product of any long-lived radioactive nucleus, show that these data are consistent with a common primordial ratio $^{87}Sr/^{86}Sr$ and a common age for all these meteorites and find that age.

X(10)

The Orion nebula is 1600 light years away from the Sun and is completely ionised. Its average temperature is 8500 K and its electronic density is 2000 cm^{-3}. Its diameter is estimated to be 1.6 light year (one light year is 365 light days).

a) Suppose that nebula is made of non collisional plasma (a good approximation), what is the cutoff frequency f_p (in Hz) for electromagnetic waves propagating in this nebula.

b) Two light pulses, P_1 and P_2, with carrier frequencies f_1=15 GHz and f_2 =20 GHz go through the nebula. If P_1 and P_2 come into the nebula at the same time, what will be the delay (in seconds) between the two signals when they come out of the nebula. (Be more intelligent than your calculator; the binomial approximation could be useful...)

c) Refer to b). At the carrier frequency f_1 , what is $\Delta\varepsilon_r (= \varepsilon_{rp} - 1)$, the difference between the relative permitivity of the plasma and that of the vacuum.