1998 CAP Undergraduate Prize Examination

Wednesday, February 11

from 2:00 to 5:00

Completed examination booklets should be sent by Department Chairmen to:

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Calculators are permitted.

Each question is to be written in a separate book with the number of the problem, the name of the candidate, and the name of the university indicated clearly on the first page.

The candidates may attempt as many questions as possible in whole or in part. It is not expected that you will complete them all.

Each question holds the same value.

1. Assume that due to global warming most of the ice on the Earth's poles has melted which caused the level of the world's oceans to rise by 50 m. Calculate by how many seconds the period of rotation of the Earth about its axis will increase as a consequence. Regard the Earth as a sphere with radius $R = 6.37 \times 10^6$ m and moment of inertia $I = 8.11 \times 10^{37}$ kg m^2. *Hint:* the moment of inertia of a solid sphere about a diameter is $I = (2/5)MR^2$, where M is the mass of the sphere; the density of water is $\rho = 10^3$ kg m^{-3}.

2. One way to produce a very high magnetic field is by flux compression. A thin-walled collapsible conducting tube is placed coaxially inside a solenoid which provides a steady field B_o. The annular space between the tube and the solenoid is filled with an explosive, and the solenoid is reinforced externally. If the tube now implodes, shrinking in radius, an induced azimuthal current will flow, and an internal magnetic pressure $B^2/2\mu_o$ will build up until it is close to the external gas pressure.

   a) Assuming that the radius of the tube shrinks very rapidly, calculate the magnetic induction B when the radius has been reduced to R, in terms of R, R_o and B_o.

   b) Calculate the surface current density in the tube needed to achieve a field of 10^3 Teslas. $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m}$.

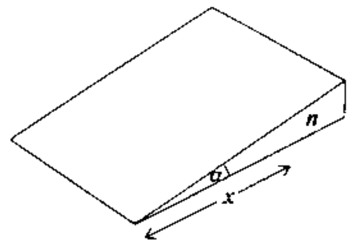
3. Reference frame S' moves in the positive x direction with respect to reference frame S. The clocks in S and S' are synchronized at $t = t' = 0$, the instant the coordinate origins O and O' of the two frames coincide. At this moment a light wave is emitted from the point $O \equiv O'$. In S it is observed that the light wave is spherical with radius $r = ct$ and is described by the equation

$$r^2 = x^2 + y^2 + z^2.$$

   a) Derive the equation representing the light wave in S'.

   b) Does the resulting equation lead to a paradox? Which basic concept should be re-examined in order to account for the apparent paradox?

4. A glass wedge of refractive index n and a small apex angle α is illuminated by a monochromatic light source of wave length λ.

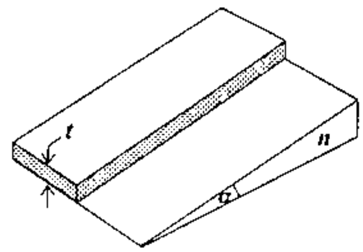


a) Show that the bright fringes when viewed vertically occur at

$$x = \frac{(m+½)\lambda}{2n\alpha}$$

where n is an integer.

b) If now a thin transparent film of uniform thickness t and of bulk refractive index n_f is deposited on half of the wedge, all fringes on this half will be shifted toward the narrow end with respect to those without the film. Given α, n, and n_f, show how the film thickness t can be determined experimentally by counting the fringes on both halves of the wedge.



5. It is known that a model of adiabatic convection currents can give a good description of the variation of the atmospheric temperature with height.

a) Considering the forces on a thin slice of thickness dh and assuming a perfect gas and an adiabatic process, calculate dT/dh (K/km). Use $\gamma = 7/5$, $g = 9.8$ m/s^2, $M = 0.029$ kg and $R = 8.2$ J/K.

b) In fact, this treatment slightly overestimates the actual temperature gradient. Suggest how the inclusion of water vapour as a component of the atmosphere could account for this discrepancy.

c) Explain how the physical model used in **a)** could also account for the particular nature of the Chinook winds.

6. A paramagnetic crystal contains N non-interacting magnetic moments μ. The crystal is placed in a heat bath at temperature T and a magnetic field B is applied to the crystal. Where appropriate use the variable $x = \mu B/kT$.

a) Calculate the partition function Z_1 for one spin.

b) Use this result to calculate the mean magnetic moment $\bar{\mu}$, hence the magnetisation M and the mean energy U.

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c) Calculate the Helmholtz Free Energy and hence the entropy of the system. Comment on the physical significance of the value of the entropy for the limits $x \ll 1$ and $x \gg 1$.

d) Calculate $C_V(T)$. Sketch the form of $C_V(T)$ and explain the physical significance of any anomalous behaviour.

e) Suggest how the results of **(b)** could be applied to low temperature thermometry.

7. The positron has the same mass as the electron, but opposite charge and spin magnetic moment. If you replace the proton in a hydrogen atom by a positron, you get a positronium atom.

a) The ground state of the atom consists of two hyperfine components, $^1S_0$ and $^3S_1$, split by a very small energy difference. Which level lies lower? Explain.

b) What is the binding energy of the ground state?

c) A positronium atom at rest in $^1S_0$ state decays to 2 γ-rays. What is their energy and relative direction?

8. A harmonic oscillator consists of a particle of mass m moving along the x-axis in a potential $V(x) = \frac{1}{2}m\omega^2 x^2$. The particle has charge e and is subjected to a perturbing electric field E in the x-direction.

a) What are the energies of the unperturbed states (before the electric field is switched on)?

b) To second order in E, what is the energy of the n-th excited state?

c) To first order in E, what is the wave function of the n-th excited state?

d) Find the exact energy eigenvalues and the exact wave functions in the presence of the electric field and compare with the results obtained in (b) and (c).

Possible useful formulae:

$$\left|\psi_n^{(1)}\right\rangle = \left|\psi_n^{(0)}\right\rangle + \lambda \sum_{k \neq n} \frac{V_{kn}}{E_n^{(0)} - E_k^{(0)}} \left|\psi_k^{(0)}\right\rangle$$

$$E_n^{(2)} = E_n^{(0)} + \lambda V_{nn} + \lambda^2 \sum_{k \neq n} \frac{V_{kn} V_{nk}}{E_n^{(0)} - E_k^{(0)}}$$

$$\left\langle \psi_k \middle| x \middle| \psi_n \right\rangle = \sqrt{\frac{\hbar}{2m\omega}} \left(\sqrt{n+1}\, \delta_{k,n+1} + \sqrt{n}\, \delta_{k,n-1} \right)$$

9. The general solution of the wave equation describing the displacements of a stretched string of length L, in the absence of external forces, is

$$u(x,t) = \sum_{n=1}^{\infty} \left(A_n \cos \omega_n t + B_n \sin \omega_n t \right) \sin k_n x$$

where $\omega_n = n\pi c/L$ and $k_n = n\pi/L$; t is the time and x the space coordinate.

a) Find the complete solution by determining A_n and B_n when the initial displacement $u(x,0)$ is zero and the initial velocity is $v_0 \cos\left[\pi\left(\frac{x-\xi}{d}\right)\right]$, for $-\frac{d}{2} < x - \xi < \frac{d}{2}$, $d < L$, $\xi < L$ and zero otherwise.

b) As a function of time, the total energy of an undamped vibrating string is given by

$$E(t) = \int_0^L \frac{\rho}{2}\left[u_t^2(x,t) + c^2 u_x^2(x,t)\right] dx,$$

where ρ is the density; u_t and u_x denote the derivatives of u with respect to t and x, respectively. The first term is the kinetic energy and the second one the potential energy. For the displacement of $u(x,t)$ corresponding to an initial velocity v_0 and to an initial position u_0, evaluate $E(t)$ and show that it is conserved, i.e., that it is independent of time. *Hint:* Express the square of a sum as a double sum and show that the resulting integrals over x are proportional to δ_{nm}.

10. Explain briefly, in one paragraph per topic, the physical principles involved in any FOUR of the following:

a) Bipolar vs. Field-Effect Transistor.

b) Synchrotron Radiation.

c) Black Hole.

d) Irreversibility: Macroscopic vs. Microscopic.

e) Bose-Einstein Condensation.

f) Soliton.