CAP University Prize Examination

February 5, 1992
2:00-5:00 p.m.

Return to:

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Instructions:

- The use of calculators is permitted.
- Answer each question in a separate booklet, with the question number and your name clearly indicated.
- Attempt as many questions as possible. Do not expect that you will be able to complete all the questions-- choose carefully.
- All questions are of equal value.

Useful Information:

Proton mass: .... $m_p = 1.67 \times 10^{-27} kg$

Elementary charge: ...... $e = 1.60 \times 10^{-19}\, C$

Gravitational constant: ... $G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

Permittivity of the vacuum: $\varepsilon_o = 8.85 \times 10^{-12}\, F/m$

Speed of light: .... $c = 3.00 \times 10^8\, m/s$

Boltzmann constant: ..... $k_B = 1.38 \times 10^{-23}\, J/K$

Mass of Earth: .... $M_E = 6 \times 10^{24}\, kg$

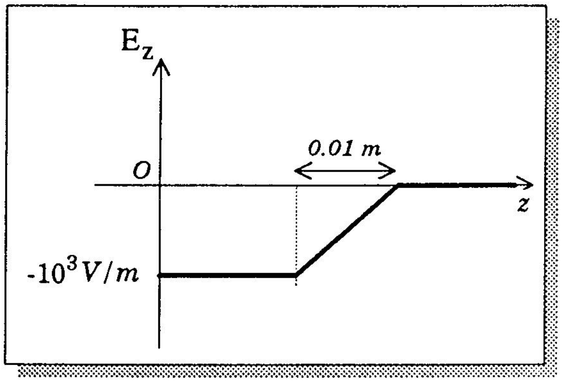
Radius of Earth: .... $R_E = 6 \times 10^6\, m$

Mass of Sun: .... $M_S = 2 \times 10^{30}\, kg$

Radius of Sun: .... $R_S = 7 \times 10^8\, m$

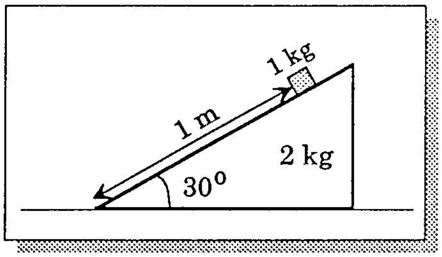
Answers in Sept 1992 issue

1. A parallel beam of protons passes through an electrostatic lens system. The fields in the lens system are cylindrically symmetric and the electric field along its z axis is shown in the figure. The proton beam travels close to the axis in the positive z direction, and is initially parallel to the axis before it reaches the "gap" region in which the electric field changes. If the protons have an energy of *1 keV* when they reach the gap, find



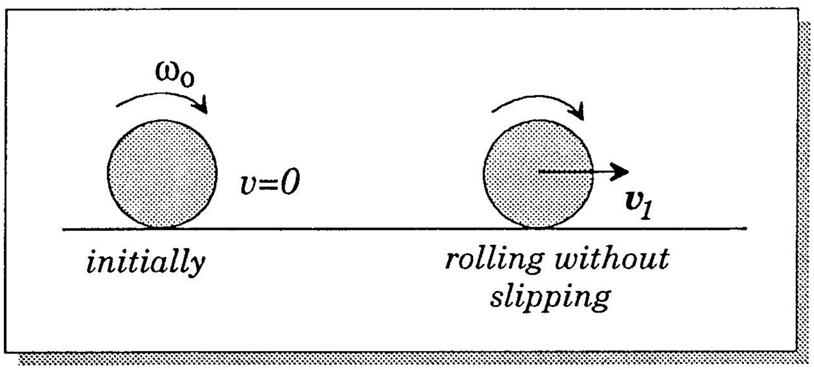
a) the speed of the proton when it exits the gap region.
b) an expression for the radial component of the electric field in the gap.
c) the time that a proton will spend in the gap.
d) the distance from the centre of the gap of the point at which the beam comes to a focus.

2. A small block of mass $m=1\ kg$ is free to slide without friction down a ramp of mass $M=2\ kg$ which moves without friction on a horizontal surface. Both the block and the ramp are initially at rest. The ramp is *1 m* long and makes an angle of 30° with the horizontal. The small block is released from the top of the ramp. Find



a) the magnitude of the force with which the block presses down on the ramp.
b) the acceleration of the ramp.
c) the time for the small block to slide down the ramp.
d) the speed of the smaller block when it reaches the foot of the ramp relative to a stationary observer.

3. A wheel of mass m, radius a and moment of inertia I is spinning at an angular velocity ω_0. It is lowered onto a flat, level surface and released with no initial (linear) velocity. The coefficients of static and kinetic friction, μ_s and μ_k, are arbitrary functions of distance along the path of the wheel. Eventually the wheel rolls without slipping and moves at a velocity v_1.



(... continued in next page)

a) Find v_1.
b) What fraction of the original kinetic energy remains ?
c) Is angular momentum conserved in this problem ? Explain.

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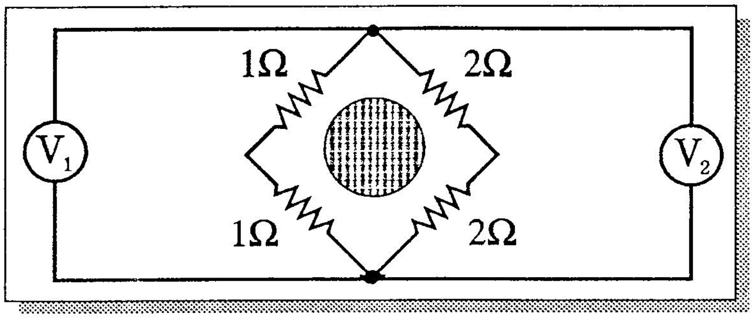
4. Dr. Frankenstein's "bridge" is illustrated below. This is not really a bridge, simply a loop of four resistors linked by a region of changing magnetic flux (shaded area in the figure). Inside the region, the flux is given by

$$\Phi = -\sin(120\,t)$$

Outside, the flux is everywhere zero. Dr. Frankenstein has two AC voltmeters to monitor the experiment, as shown. They have very high input impedances, so they draw negligible current. They measure the root mean square (*r.m.s.*) voltage across their terminals.



a) What current flows around the loop of resistors ?
b) What do the voltmeters V_1 and V_2 read ?

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5. In special relativity the acceleration of an object depends on the reference frame of the observer. Consider two inertial reference frames S and S' where S' moves to the right of S with speed v. The relation between the speeds u and u' of an object measured by S and S' is

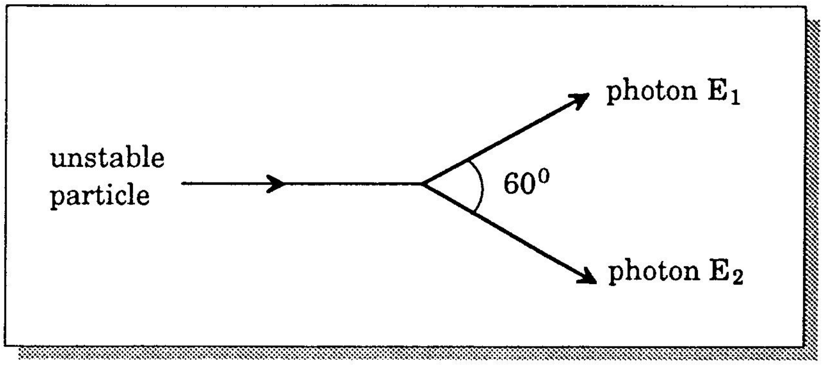
$$u = \frac{u' + v}{1 + \frac{vu'}{c^2}}$$

a) Show that the relation between accelerations $a' = \frac{du'}{dt'}$ and $a = \frac{du}{dt}$ measured in the two frames for an object instantaneously at rest in S' is

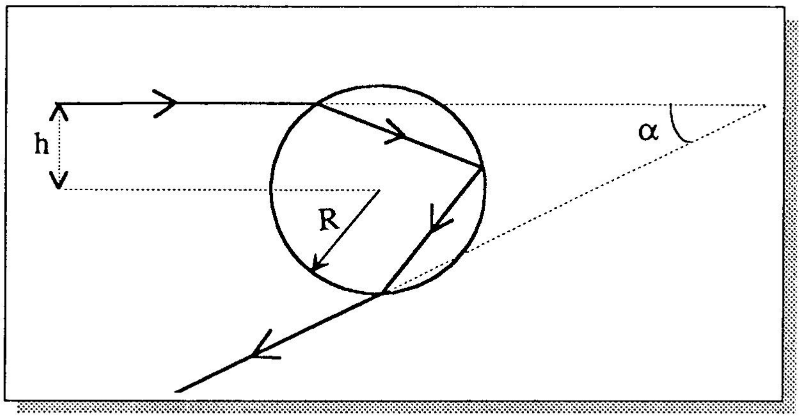
$$a' = \gamma^3 a, \qquad \gamma = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}$$

b) You have just received a job with a research team designing a spaceship for interstellar travel. To introduce you to the project your supervisor tells you that the goal of the project is to design a spaceship which will produce a uniform acceleration of $a' = 9.8\,m/s^2$ in its instantaneous rest frame. Your first task is to answer the following question: If the spaceship starts from rest in earth orbit, what would be the speed of the spaceship observed from earth after 10^8 *seconds* of earth time (about three years)?

6. An unstable particle decays in flight into two photons. If the photons are observed to have energies $E_1 = 161\,MeV$ and $E_2 = 119\,MeV$ with the indicated geometry, find the rest mass of the unstable particle.



7. A ray of light enters a transparent spherical drop of liquid, radius R and index of refraction n, at a distance h from its centre and is back-scattered as shown in the figure. Find the angle α between the incident and scattered ray in terms of h, R and n. What is the significance of the maximum value of α if light is equally likely to arrive at all possible values of h?



8. Consider a particle of mass m moving in a one dimensional infinite square well

$$V(x) = \begin{cases} \infty & \text{for } -\infty < x < -\frac{L}{2} \\ 0 & \text{for } -\frac{L}{2} \le x \le \frac{L}{2} \\ \infty & \text{for } \frac{L}{2} < x < \infty \end{cases}$$

Notice that the potential is infinite everywhere except a region of length L, where it is zero. The normalized wave functions and corresponding energies for the ground and first excited state of the particle are:

$$\Phi_1(x,t)=\sqrt{\frac{2}{L}}\cos\left(\pi\frac{x}{L}\right)\exp\left(-iE_1\frac{t}{\hbar}\right), \quad E_1=\frac{h^2}{8mL^2}$$

$$\Phi_2(x,t)=\sqrt{\frac{2}{L}}\sin\left(2\pi\frac{x}{L}\right)\exp\left(-iE_2\frac{t}{\hbar}\right), \quad E_2=4E_1$$

According to the principle of superposition, the linear combination

$$\Psi(x,t)=C\,\Phi_1(x,t)+C\,\Phi_2(x,t)$$

represents a state of the system.

a) Find the value of the constant C.

b) Is Ψ a stationary state?

c) Do the eigenfunctions Φ_1 and Φ_2 have definite parity?

d) At $t>0$ we perform an ensemble measurement of the total energy of the particle whose state is represented by $\Psi(x,t)$. Find the average energy one would obtain in such a measurement. Does the average value of energy depend on time?

e) Is the momentum of the particle a constant of motion?

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9. Consider an electron of mass m moving in a one-dimensional silicon wire of length L. The electron sees a constant potential within the wire. The wave functions of the electron satisfy the periodic boundary conditions at the end points of the wire.

$$\Psi(x=0)=\Psi(x=L)$$

a) Write the time-dependent Schrödinger equation for the electron in the wire.

b) Find the eigenfunctions and eigenvalues of the electron in the wire.

c) Show that the eigenfunctions of the electron form an orthonormal set.

d) Is the eigenfunction of the electron a bound or unbound state? Why?

e) Show that momentum is quantized.

10.

a) Estimate the pressure in the centre of the planet earth. *Hint: the easiest method is based on a dimensional analysis noting that the equation can only depend on the gravitational constant, mass and radius. The more complete method is based on the fact that each shell causes pressure on the lower layer, attracted by the remaining material below.*

b) Assuming the sun to be an ideal gas of atomic hydrogen, estimate the pressure, temperature and density in the centre of the sun.

11. An ideal gas with an adiabatic coefficient $\kappa > 1$ undergoes the following simple cyclic process.

1	$Z_1(p, V_1, T_0) \rightarrow Z_2(p, V, T)$	expansion
2	$Z_2(p, V, T) \rightarrow Z_3(p_3, V, T_0)$	cooling
3	$Z_3(p_3, V, T_0) \rightarrow Z_1(p, V_1, T_0)$	compression

a) Sketch this process in a pressure-volume and an entropy-temperature diagram.
b) Calculate the work done and the amount of heat which the gas exchanges with its environment per unit mass of the gas during each of the above steps.
c) How does the internal energy change during each step?
d) Calculate the efficiency factor for this process, i.e. the ratio of the released work to the heat input during a full cycle. Compare your result to the efficiency of the Carnot process.

12. Determine the lattice energy of a one-dimensional chain of alternating (singly-ionized) positive and negative ions with spacing r_o. Express your answer in terms of the Madelung constant μ:

$$E_{attract} = -\mu \frac{e^2}{4\pi\varepsilon_o r_o}$$

Apply this result to a "one-dimensional" NaCl crystal (density $2.165\ g/cm^3$, mass number of Na = 23, mass number of Cl = 35). Comment on any difference between your result and the lattice energy of *-8.8 eV* for a three-dimensional NaCl crystal.
Hint: Use the Taylor series for $\ln(1+x)$ *to simplify your result.*

13. You want to determine the concentration of the charge carriers, the mobility of the charge carriers and their electrical sign in a semiconductor. Describe the minimum set of experiments you need to perform to get this information.

14. The semi-empirical mass formula for the binding energy *B(A,Z)* of a nucleus with *Z* protons and *A-Z* neutrons is

$$B(A,Z) = aA - bA^{2/3} - dZ^2A^{-1/3} - \frac{s(A-2Z)^2}{A} .$$

In MeV units the values of the constants in this expression are $a = 15.8$, $b = 18.3$, $d = 0.71$ and $s = 23.2$.

a) Use *B(A,Z)* to deduce the *Z* value corresponding to the most stable isobar with mass number *A=208*. Neglect the mass difference between the neutron and proton.
b) If *Z* is now put equal to $A/2$ in the above expression, prove that the binding energy per nucleon, B/A, has a maximum value at a particular value of *Z*. Estimate this value of *Z*.