CAP UNIVERSITY PRIZE EXAMINATION

JANUARY 31, 1990

2 p.m. to 5 p.m.

COMPLETED EXAMINATION BOOKLETS SHOULD BE SENT
BY DEPARTMENT CHAIRMEN TO:

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DEPARTMENT OF PHYSICS
LAKEHEAD UNIVERSITY
THUNDER BAY, ONTARIO
P7B 5E1

INSTRUCTIONS AND INFORMATION

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1. The use of calculators is allowed.
2. Attempt as many questions as you can in whole or in part.
3. ANSWER EACH QUESTION IN A SEPARATE BOOKLET, with the question number and your name on the outer page of each booklet.
4. All questions are of equal value, but not of equal difficulty.
5. Universal gas constant R = 1.9864 Cal/deg.mole

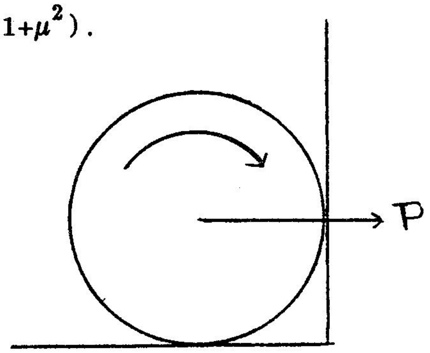
   1 mole of air has a mass of 29 g

   1 mole of helium gas has a mass of 4 g

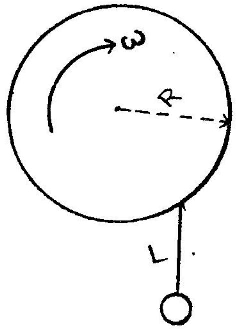
   1 mole of gas at 0^0C and 1 atm occupies (2.2414) 10^4 cc

1. A uniform circular cylinder, of weight W and radius a, rests on level ground and is pressed against a rough vertical wall by a horizontal force P through its axis. The contacts of the cylinder with the wall and the ground are equally rough, with coefficient of friction μ. Show that the couple G necessary to rotate the cylinder about its axis in the sense shown, with slipping taking place at both the contacts, is:

$$G = a\mu[(1+\mu)W + (1-\mu)P]/ (1+\mu^2).$$



2. A pendulum clock for use on a gravity-free spacecraft was constructed. The design consisted of a simple pendulum formed by a massless rod of length L. A mass M was attached to one of its ends while the other end was pivoted so that the simple pendulum could swing in a plane. The pivot was forced to rotate in the same plane, at angular frequency ω in a circle of radius R, thereby creating an artificial gravity for the pendulum. Show that this pendulum simulates the simple pendulum with acceleration $g = \omega^2 R$ for all values of L and all amplitudes of the oscillations.



3. There is a spaceship shuttle service between Earth and Mars. Each of the spaceships involved is equipped with two identical lights of proper wavelength λ_0, one at the front and one at the rear. The spaceships normally travel at a speed v, relative to the Earth, with the result that the headlight of a spaceship approaching the Earth appears green (λ_1 = 5000 A) while the tail-light of a departing spaceship appears red (λ_2 = 6000 A).

(a) Determine v and λ_0

(b) If a Mars-bound spaceship A accelerates to overtake another Mars-bound spaceship B ahead of it, at what speed must it travel, relative to earth, so that the tail-light of the spaceship B appears to it as a headlight (5000 A green)?

4. Obtain an expression for the resolving power ($\lambda/\Delta\lambda$), of Michelson's interferometer used as a scanning interferometer with a total mirror displacement Δx.

5. Consider a balloon filled with 1 mole of helium gas at atmospheric pressure and at temperature 25^{0}C. What is the diameter of the balloon? What is its buoyancy? Compute the internal energy and its specific heat. Assume that the gas is ideal.

6. An electrolyte cell has an equation of state $E = E_0 - \alpha(T-T_0)$ and internal energy $U = U_0 + (E - \alpha T)Z + C_z(T-T_0)$ where E is the emf of the cell, Z is the charge on the cell, T is the absolute temperature and U_0, α, C_z, and T_0 are constants.

a) The cell performs n Carnot cycles where heat Q_H flows out of the heat bath at temperature T_H and heat Q_C flows into the cold bath at temperature T_C during each cycle. What is the entropy change of the hot bath, the cold bath, the cell and the universe?

b) Charge ΔZ flows out of the cell, kept under isothermal conditions at temperature T_1, into a thermally insulated resistor with heat capacity C_p and initially at temperature T_1 also. What is the entropy change of the universe in terms of ΔZ?

7. Two parallel surfaces are maintained at temperatures T_1 and T_2 with $T_1 > T_2$. They are allowed to exchange energy. The emissivities of the surfaces are, respectively, ϵ_1 and ϵ_2. Derive the expression for the radiative flux of energy from the hot to the cold surface. To provide a radiation shield, a thin metal screen of emissivity ϵ is placed between the two surfaces and is allowed to attain its own temperature. Estimate the factor by which the energy exchange is reduced by the screen, if we assume $\epsilon_1 = \epsilon_2 = \epsilon$.

8. An electron of charge e is moving between two coaxial cylinders of radii a and b (a<b). There is a constant magnetic field through the region r < a, but it does not penetrate into the region a < r < b, where the electron is confined. Ignore r and z motions, and use the Hamiltonian:

$$H = (1/2m)[P_\varphi - (e/c)\, A_\varphi]^2$$

where

$$\frac{\partial A_\varphi}{\partial \varphi} = 0 \quad \text{and} \quad P_\varphi = -\frac{i\hbar}{a}\frac{\partial}{\partial \varphi}$$

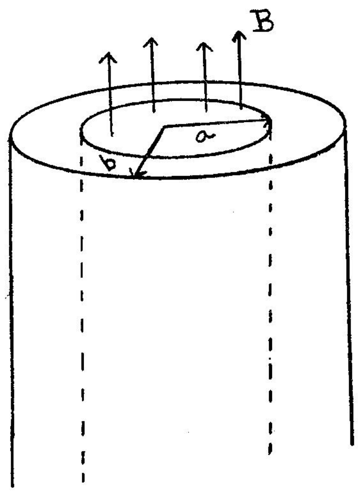
to obtain the Schrodinger equation. Show that the energy eigenvalues are

$$E_n = \frac{\hbar^2}{2ma^2}[n + (\phi_0/\phi)]^2$$

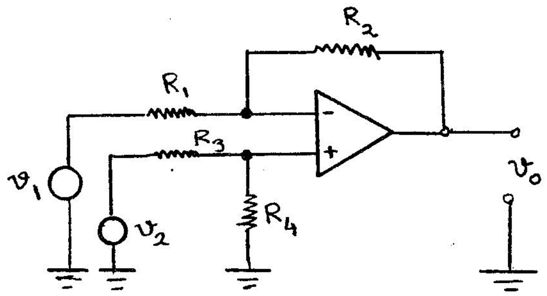
where

$$\phi_0 = (2\pi c\hbar/e) \text{ and } \phi = 2\pi a A_\varphi.$$

ϕ is the flux trapped in the inner cylinder.



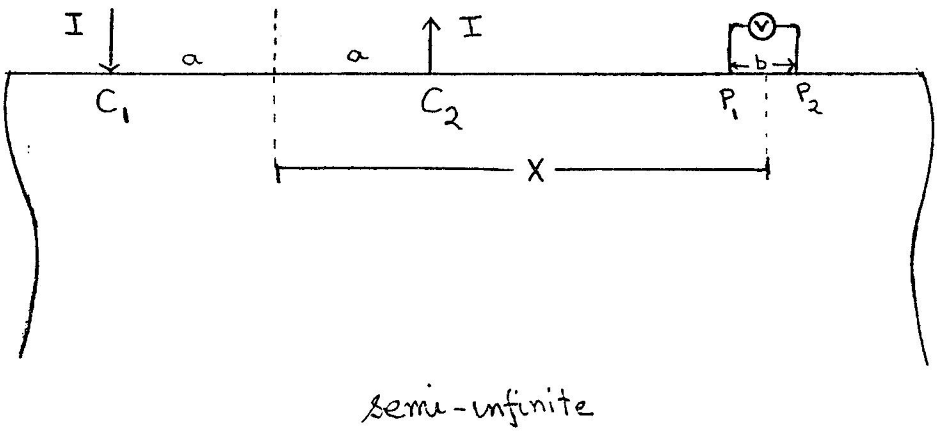
9. The differential amplifier can be used for measuring deviation. If we set, in the circuit below, $R_3 = R_4$ and $R_2 = R_1 + \Delta R$ and connect the two inputs together to the same voltage v (i.e. $v_1 = v_2$), express v_0 in terms of ΔR.



10. Resistivity anomalies in the ground can be located as in the figure below. The current I flowing between electrodes C_1 and C_2 establishes an electric field in the ground and one measures the voltage between a pair of electrodes P_1 and P_2 separated by distance b. With b << a , (V/b) is equal to E at position x. Show that if the ground conductivity is uniform and equal to σ

$$(V/b) = -\frac{2axI}{\pi\sigma(x^2-a^2)^2}$$

for infinitely small electrodes.



11. Consider a plane Z = 0 which divides two semi-infinite regions; the Z > 0 region is maintained at potential $V = V_1$, and the Z < 0 and Z = 0 is maintained at potential $V = V_2$. A beam of particles is incident on the surface with probability current $\hbar \underset{\sim}{k}_1 |A^2|/m$ particles/cm^2-s. Particles in the reflected and transmitted beams have respectively momemtums $\hbar \underset{\sim}{k}_2$ and $\hbar \underset{\sim}{k}_3$. The vectors $\underset{\sim}{k}_1, \underset{\sim}{k}_2$ and $\underset{\sim}{k}_3$ are all in one plane. Calculate the reflection coefficient R and the transmission coefficient T. Compare the results with the laws of reflection/refraction in optics.

12. A homogeneous mixture of enriched uranium and carbon forms the core of a simple nuclear reactor. Suppose that

(1) the core is a cube of side L;

(2) the neutrons emitted by the uranium travel with an average speed v and a mean free path λ;

(3) neutrons are captured, on an average, after N collisions;

(4) for each neutron captured K new neutrons are emitted.

Obtain the rate of change of neutron density, n, in the core. Estimate the size of the pile required to achieve a chain reaction, given that $\lambda = 10$ cm, $K = 1.04$ and $N = 100$.
