CAP UNIVERSITY PRIZE EXAMINATION

February 8, 1989

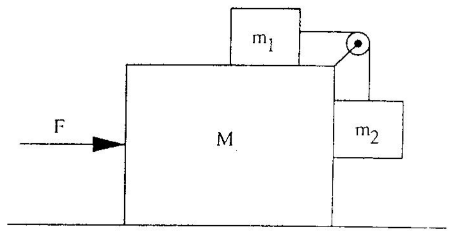
2:00 - 5:00 P.M.

INSTRUCTIONS

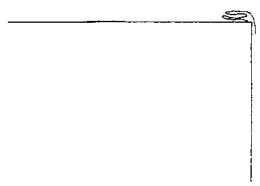
1. The use of calculators is allowed.
2. Do as many questions as you can, in whole or in part. It is not expected that you will complete all!
3. Answer each question in a separate booklet, with the question number and your name on the outside of each booklet.
4. All questions are of equal value, but are not of equal difficulty.

QUESTIONS

1. A force $\vec{F}$ is applied to M in the system shown here. Assuming that the coefficient of friction between m_1 and M and between m_2 and M is μ ($\mu < m_2/m_1$; $\mu < m_1/m_2$) but that there is no friction between M and the ground, find the range of forces $\vec{F}$ which will keep m_1 stationary with respect to M.

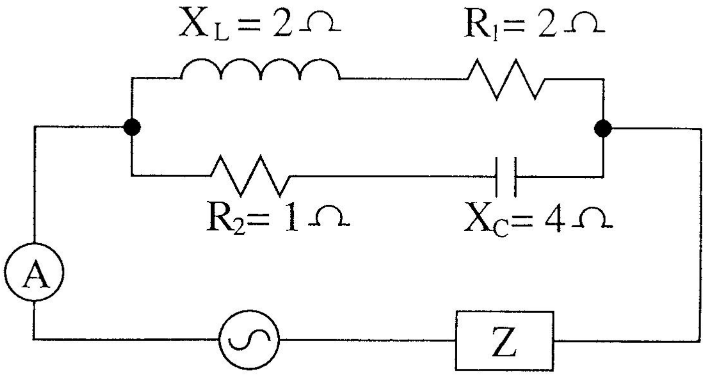


2. A length of rope is piled up at the edge of a frictionless surface. An end section of negligible length is initially at rest, and hangs over the edge. Assuming the rope is of uniform density, and that it unravels itself smoothly as it slides over the edge, calculate the acceleration of the endpoint.



3. Derive the shape of the mirror surface that has the property that rays from a particular point on the axis are reflected into a parallel beam.

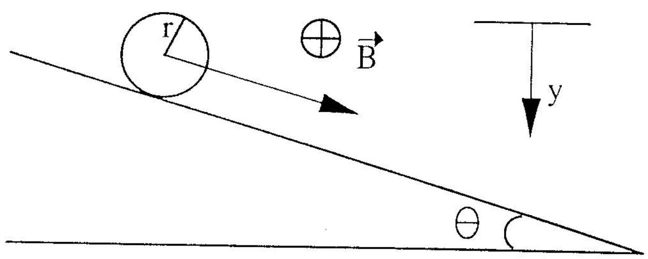
4.



(a) In the circuit shown, what value of the impedance Z will give rise to the largest reading of the ammeter A?

(b) What kind and size of circuit element(s) must be used to provide this value of Z for a circuit operating at 60 Hz?

5.



A ring rolls down an incline in a magnetic field $\vec{B}$ perpendicular to the plane of the ring. If the <u>electrical</u> resistance of the ring is R and its mass is m, and the magnetic field varies linearly with y ($B = B_o y$), find the terminal velocity of the ring.

6. A positively charged muon forms a hydrogen-like atom (muonium) with an electron. The muon decays into a positron, an electron neutrino and a muon antineutrino which rapidly leave the region where the muonium existed. What is the average value of the kinetic energy of the remaining electron if the decay occurred when the muonium was in the 2S state? The electron wavefunction is

$$\psi_{2s} = \left\{1/\sqrt{8\pi\, a_o^3}\right\}(1 - r/2a_o)\ \exp[-r/2a_o].$$

a_o is the Bohr radius. For the hydrogen atom,

$$E_n = -\, e^2/8\pi\varepsilon_o a_o n^2.$$

<u>Express an answer in eV.</u>

Note: $\int_0^\infty x^n\, e^{-ax}\, dx = n^1/a^{n+1}$.

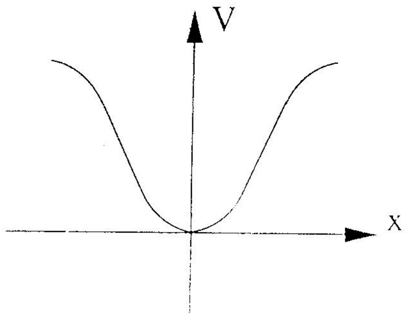
7. (a) For a particle moving in space, show that the wavefunction

$$\psi_{\ell m}(x,y,z) = x^2 + y^2 - 2z^2$$

represents a simultaneous eigenstate of the total angular momentum operators J^2 and J_z.

(b) Find a function with the same eigenvalue for J^2 and having the maximum possible value for J_z also.

8. A particle moves in a symmetrical one-dimensional potential, as illustrated:



In <u>each</u> of the following 3 cases, state (giving a reason for your answer in each case) whether the perturbation correction to the energy does one of the following:

[A] - raises the energy in first order; <u>or</u>

[B] - raises the energy only in second order; <u>or</u>

[C] - lowers the energy in first order; <u>or</u>

[D] - lowers the energy only in second order; <u>or</u>

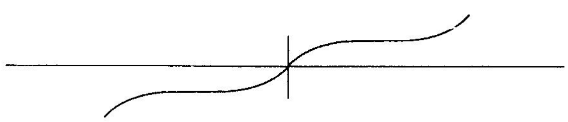
[E] - doesn't change the energy.

(a) Perturbing potential: 

Assume the system is in the <u>ground</u> state.

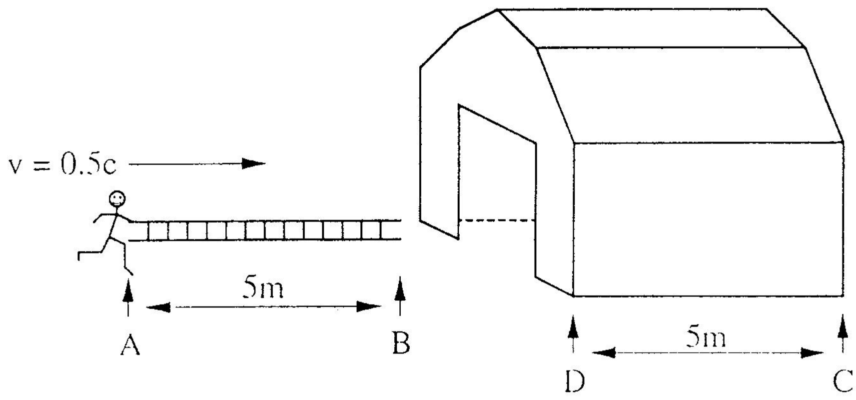
(b) Perturbing potential: 

In this case assume the system is in the <u>first excited</u> state.

(c) Perturbing potential: 

Assume the system is in the <u>ground</u> state.

9. An athlete runs with speed $\upsilon = 0.5c$ towards the open door of a barn holding a long ladder horizontally at one end (see diagram below). The ladder is exactly 5 m long in its rest frame, and the barn is exactly 5 m long in its rest frame. According to Einstein's special theory of relativity, does the person get to the door of the barn in time to shut it behind him before the end of the ladder hits the far end of the barn? Justify your answer by illustrating the situation on a space-time diagram.



10. Consider a solid of volume V to be well approximated by an array of N independent isotropic quantum mechanical simple harmonic oscillators (force constant k, mass m) in thermal equilibrium with a heat bath at temperature T.

(a) Calculate the partition function and mean energy for this system.

(b) What is the molar heat capacity for this system? In the limit $h \rightarrow 0$, compare your answer to the one predicted by the equipartition theorem.

11. The order of levels in a nucleus consisting of a single particle outside an inert core may be described in first approximation using a simple harmonic oscillator potential plus potentials depending on the angular momentum and spin of the unpaired nucleon.

(a) Using these ideas discuss the order of levels up to the top of the s-d shell.

(b) Assume that ^{17}O consists of a single neutron outside a core of ^{16}O of total spin 0 and parity +. What are the total spins and parities of the ground state and first two excited states of ^{17}O?

(c) A Λ hyperon is a baryon of spin 1/2 and charge 0 (like the neutron), but differs from the neutron in that one of the d (down) quarks is replaced by an s (strange) quark. $^{17}_{\Lambda}O$ consists of 8 protons and 8 neutrons plus the hyperon. Assuming that the hyperon feels the same forces as does a neutron, what are the total spins and parities of the ground state and the first two excited states of the nucleus $^{17}_{\Lambda}O$?

12. (a) Derive the equation that gives the number of atoms N of a nuclide that is produced at a constant rate R and decays with a probability λ per unit time. Assume that at t=0, N=0.

(b) ^{238}U decays by emitting a series of alphas and betas through Rn to ^{206}Pb. The half lives are:

$$^{238}U = 4.5 \times 10^9 \text{ years}$$
$$^{222}Rn = 3.8 \text{ days}$$

If there is a 100 mg thin foil of U exposed to the air in a closed room, what is the equilibrium concentration of radon gas in the room? Dimensions of the room are 5.0 m x 4.0 m x 2.5 m.

(c) The concentration of radon gas can be measured experimentally by exposing a small activated charcoal canister to the air in the room for about one month. The canister is then moved to a detector and the activity measured. Describe how the radon concentration in the room can be deduced from the measured count rates on the charcoal.

THE END