# CANADIAN ASSOCIATION OF PHYSICISTS

## University Prize Examination

Tuesday, February 9th, 1982

2:00 p.m. to 5:00 p.m.

Examination Committee:

Dr. R.M. Clements
Dr. F.I. Cooperstock
Dr. G.R. Mason
Dr. H.M. Sullivan
Dr. J.B. Tatum
Dr. A. Watton
Dr. J.T. Weaver
Dr. C.-S. Wu

Return completed booklets to:

Dr. A. Watton
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Instructions:

1. Slide rules or pocket calculators without stored memory are allowed.

2. ANSWER EACH QUESTION IN A SEPARATE BOOKLET, with the question number and your name on the outer page.

3. Each question is worth 20 marks, although not all questions are of equal difficulty.

1. A building at a temperature T°K is heated by means of a Carnot refrigerator (heat pump) which uses a river at $T_o$°K as a source of heat and consumes a constant power W. The building loses heat to its surroundings at a rate $\alpha(T-T_o)$ where $\alpha$ is a constant. Show that the equilibrium temperature of the building is

$$T_{eq} = T_o + \frac{W}{2\alpha}\left\{1 + \left(1 + \frac{4\alpha T_o}{W}\right)^{\frac{1}{2}}\right\} .$$

2. Discuss briefly, and justify, appropriate methods of measuring the distance between:

   a) atoms in a crystal.

   b) two atoms in a hydrogen molecule.

   c) two points about 10 km apart on the earth's surface.

   d) the earth and the moon.

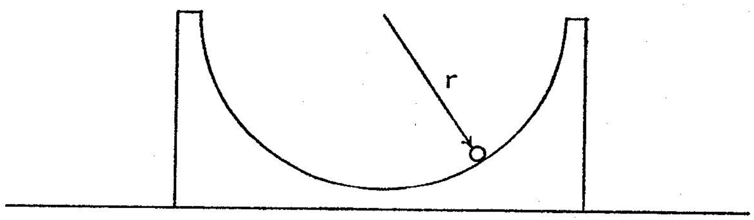
   e) the earth and a nearby star.

   Your discussion should include an explanation of the physical principles behind the methods you have chosen.

3. A semicircular trough, radius r, is cut in a block which sits on a smooth horizontal surface. A bead slides freely on the inside of the trough.

   By consideration of the Lagrangian, or otherwise, show that for small amplitudes of vibration the bead executes simple harmonic motion with period $2\pi\sqrt{\frac{Mr}{(M+m)g}}$ where M and m are the masses of the block and bead respectively.

   Neglect all frictional forces.



4. A narrow rectangular hole of proper length 10 meters is cut in the ice and a team of physicists equipped with sledge hammers are positioned along its length. A sturdy steel rod of proper length 10 meters slides along the ice towards the hole with a speed .99 c and when it is directly over the hole, the physicists simultaneously hit the rod down towards the hole (its horizontal motion is unchanged).

a) What are the lengthsof the hole and rod as perceived by the physicists? Does the rod enter the hole?

b) What are the lengths of the rod and hole as perceived in the rest frame of the rod? From this point of view, is there a contradiction? Does the rod enter the hole? If the rod does enter the hole, how does the process appear from the vantage point of the rest frame of the rod? Discuss briefly.

5. a) Use the simple Bohr model to derive the formula for the energies of the states of a hydrogen-like atom:

$$E_n = -\tfrac{1}{2}mc^2 \frac{Z^2\alpha^2}{n^2}$$

where $\alpha$ = fine-structure constant = $(137.036)^{-1}$.

b) On the basis of this model, calculate the ground-state energy (in eV) of an "exotic" atom consisting of a positive pion ($\pi^+$, rest mass energy 139.6 MeV) and a negative muon ($\mu^-$, rest mass energy 105.7 MeV).

c) What is the name of the best quantum mechanical equation to describe the $\pi^+\mu^-$ system? (Justify your answer using one or two sentences.)

d) What are the possible values for the magnitude of the total angular momentum of the $\pi^+\mu^-$ system if the orbital-angular-momentum quantum number is 1?

e) Assuming that a sufficient number of $\pi^+\mu^-$ exotic atoms could be produced, what type of detector might best be used to observe the $K_\alpha$ transition? (Justify your answer using one or two sentences.)

6. A 6 watt beam of light of wavelength $\lambda = 3 \times 10^{-7}$ m is totally reflected by an aluminum mirror.

a) How many photons are reflected from the mirror every second?

b) What is the force exerted on the mirror?

c) If the mirror were covered by lamp-black so that the beam of light is absorbed, what would be the force exerted on the mirror?

Data: $h = 6.63 \times 10^{-34}$ J•sec.

7. a) From the operator representation of linear momentum

$$p_x = -i\hbar \frac{\partial}{\partial x}$$

show that

$$[p_x, F(x,y,z)] = -i\hbar \frac{\partial F}{\partial x} .$$

b) Write down the expression for the expectation value of a dynamical variable A in a state described by wave function $\psi(x,t)$, and show that (by use of the Schrödinger equation)

$$\frac{d}{dt}\langle A\rangle = \frac{1}{i\hbar}\langle [A,H]\rangle + \langle \frac{\partial A}{\partial t} \rangle ,$$

where H is the Hamiltonian.

c) For a particle with electric charge q in a uniform electric field E, show that the expectation value of the linear momentum, $\langle p\rangle$, of a wave packet changes in time in exactly the same manner as p of a classical particle.

8. Answer either part A or part B.

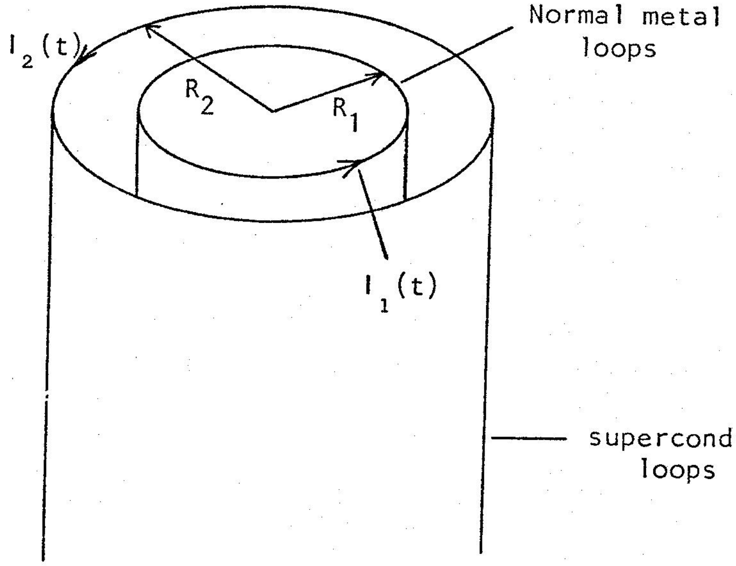
A. An inductive load draws 10 A from a 120 V, 60 Hz power source.

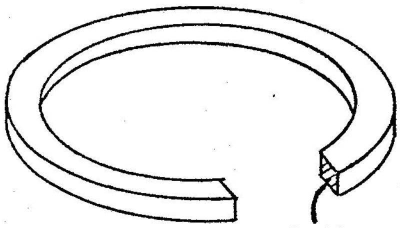
i) If the circuit power factor is ½ what is the average power dissipated in the load?

ii) If the power factor is raised to unity by connecting a capacitor in parallel with the load, what value of capacitance is required?

iii) After correcting the power factor as in ii), draw a phasor diagram, roughly to scale, showing the magnitudes and phases of the currents in the load, the capacitor and the line, in relation to the line voltage.

8. B. Consider two very long coaxial solenoids of length L, as shown below. The mean radius and cross-sectional area of the solenoids are $R_1$ and $S_1 = \pi R_1^2$ for the inner one and $R_2$ and $S_2 = \pi R_2^2$ for the outer one, respectively.




Each solenoid consists of N identical closed loops, made from wire of square cross-section and of side $L/N \ll R_1$ or $R_2$, as shown.

The inner solenoid is made of normal metal, of finite conductivity $\sigma$. The outer solenoid is made of superconducting material, of infinite conductivity

By assumption, each loop of the inner solenoid carries a current $I_o$ at time $t = 0$ but there is no current in the outer loops at that same time. Furthermore, there are no batteries involved in the problem, for $t > 0$, and end effects are neglected.

a) Let $\Phi_2(t)$ be the magnetic flux linking one loop of the outer solenoid at time $t > 0$. Describe the time-evolution of $\Phi_2$, for $t > 0$.

b) Show that the current $I_1(t)$ in one loop of the inner solenoid decays with a time constant

$$\tau = (1 - S_1/S_2)\sigma\mu_o R_1 L/2N \ ;$$

$\mu_o$ is the vacuum permeability.

END