CANADIAN ASSOCIATION OF PHYSICISTS

UNIVERSITY PRIZE EXAMINATION

Tuesday, January 29th, 1980

2:00 p.m. to 5:00 p.m.

Completed examination booklets should be sent to:

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Examination Committee: Dr. G. Beaudet
Dr. J. Brebner
Dr. P. Depommier
Dr. G. Fontaine
Dr. P. Taras

Instructions

ANSWER EACH QUESTION IN A SEPARATE BOOKLET .

1. Write your name and question number on the cover of each booklet.
2. Slide rules or pocket calculators only are allowed.
3. All the questions have the same weight.
4. Answer as many questions as you can, but keep in mind that only the very best students are expected to complete successfully more than six questions.

Useful relations

$$V = -\int \underset{\sim}{E} \cdot d\underset{\sim}{\ell} \qquad \int \underset{\sim}{E} \cdot \underset{\sim}{n}\, da = Q_{int}/\epsilon_0$$

$$E = E_0 / k \qquad \underset{\sim}{J}_f = \sigma \underset{\sim}{E}$$

$$U = Q^2 / 2C \qquad \frac{E}{H} = \frac{\omega\mu}{K}$$

$$\nabla^2 \underset{\sim}{E} - \epsilon\mu \frac{\partial^2 \underset{\sim}{E}}{\partial t^2} = \underset{\sim}{\nabla} \left(\frac{\rho_{free}}{\epsilon}\right) + \mu \frac{\partial \underset{\sim}{J}_f}{\partial t}$$

$$\nabla^2 \underset{\sim}{B} - \epsilon\mu \frac{\partial^2 \underset{\sim}{B}}{\partial t^2} = - \mu \underset{\sim}{\nabla} \times \underset{\sim}{J}_f$$

Mass excess: $[M-A]_{^7Li} = 14909$ keV ; $[M-A]_p = 7289$ keV ; $[M-A]_\alpha = 2425$ keV.

1. a) Using Huygens principle, calculate the diffraction pattern produced upon illumination of a rectangular aperture in an opaque screen. Use the Fraunhofer approximation to evaluate the appropriate integrals.

   b) Using these results, give a qualitative description of the diffraction pattern. For example, make a drawning indicating the position and relative amplitude of several maxima.

   c) Show that, if one of the dimensions of the rectangular aperture becomes much smaller than the wavelength, a well-known result is obtained.

2. Two identical perfect gases are confined at the same pressure to two vessels of volume V_1 and V_2 at temperatures T_1 and T_2 and are made up of N and 3N particles respectively. The vessels are then connected together and allowed to come to equilibrium with no heat loss to the surroundings. Determine the change in entropy of the system.

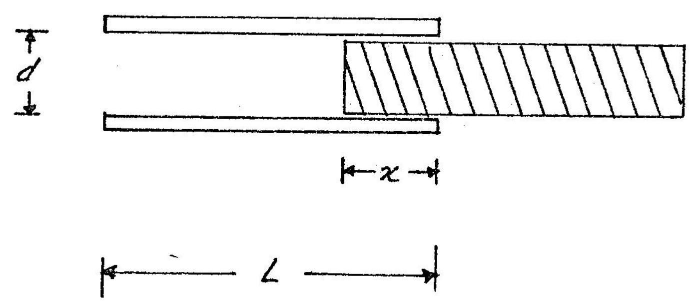
3. A parallel plate capacitor has plates of length L, width b, separated by a distance d. The capacitor is isolated and carries a total charge Q on one plate and - Q on the other plate.

   A large sheet of dielectric with a thickness slightly less than d, and a dielectric constant k, is inserted between the plates up to a distance x.

   a) Compute the capacity of the system <u>using the relation</u> Q = CV

   b) Explain intuitively why a force should act on the dielectric. What is its direction? Make a drawing.

   c) Compute that force.



4. a) Use the wave equations for $\underset{\sim}{E}$ and $\underset{\sim}{B}$ in a homogeneous medium characterized by the permittivity ϵ and the permeability μ, to show that in good conductors $\left(Q = \frac{\omega\epsilon}{\sigma} \leq \frac{1}{50}\ ;\ \rho_{free} = 0\right)$, the wave number is given by

$$K = \left(\frac{\omega\sigma\mu}{2}\right)^{1/2}(1-j) = (\omega\sigma\mu)^{1/2}\, e^{-j\pi/4}$$

and the attenuation distance δ and the radian length $\lambdabar$ by

$$\delta = \lambdabar = \left(\frac{2}{\omega\sigma\mu}\right)^{1/2}.$$

Assume that $\underset{\sim}{E} = E_0 \exp\ j(\omega t - Kz)\underset{\sim}{i}$.

b) Show that for good conductors

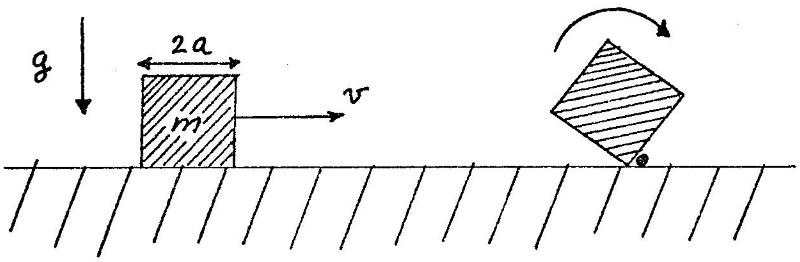
$$\underset{\sim}{E} = E_0\, e^{-z/\delta} \cos(\omega t - z/\delta)\underset{\sim}{i}$$

$$\underset{\sim}{H} = H_0 e^{-z/\delta} \cos\left(\omega t - \frac{z}{\delta} - \frac{\pi}{4}\right)\underset{\sim}{j}$$

What is the value of H_0 in terms of E_0, σ, ω and μ?

c) In good conductors, what is the attenuation factor of an electromagnetic wave and what is the phase angle between the vectors $\underset{\sim}{E}$ and $\underset{\sim}{H}$?

5. A cube of uniform density and edge 2a is sliding with velocity v on a smooth horizontal table. Its leading edge is suddenly brought to rest by a small ridge on the table parallel to the lower leading edge of the cube, and perpendicular to the direction of motion of the cube. Determine the minimum value of v for which the cube will topple over.

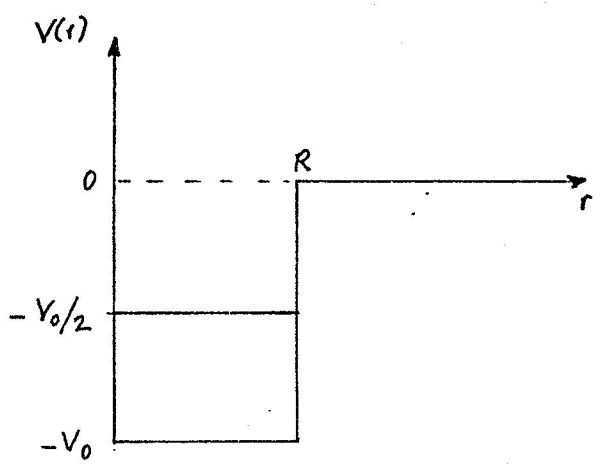


6. Consider the potential well $V(r)$ defined as follows

$$V(r) = -V_0 \quad \text{for} \quad 0 \le r \le R$$

$$V(r) = 0 \quad \text{for} \quad R < r < +\infty$$

(r is the distance from the origin of the coordinate system).



One is interested only in the states with orbital momentum $\ell = 0$ of a particle of mass m acted upon by the potential.

a) What relation must exist between V_0 and R in order for the potential to have only one bound state of energy $E = -V_0/2$?

b) Give the full expression for the wave function $\psi(r,\theta,\varphi)$ of that state.

c) Compute, for that state, the probability of finding the particle inside the well.

7. Given the reaction

$$^7\text{Li}(p,\alpha)\alpha$$

a) Compute the Q-value of the reaction.

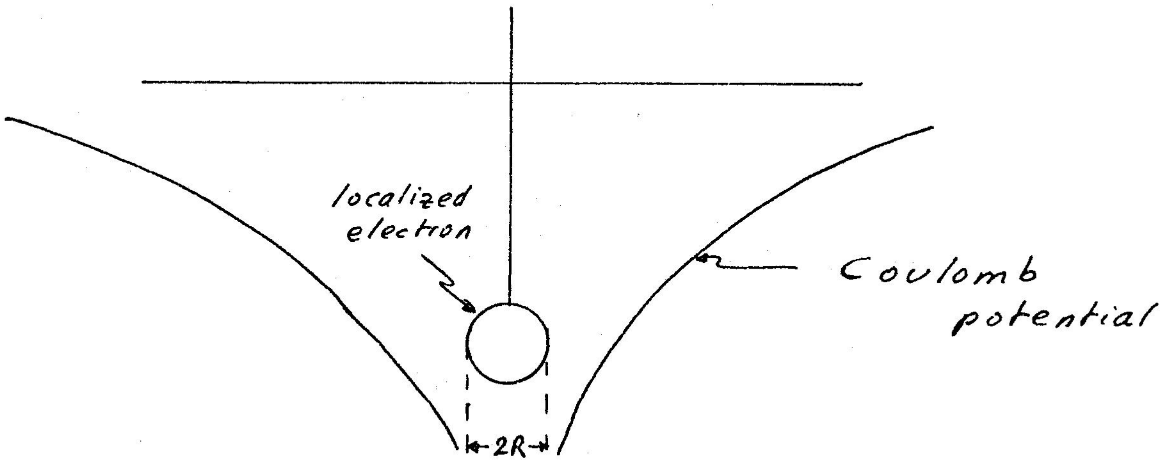
b) Compute the minimum energy of the incident particle required to produce this reaction.

c) Assuming that the incident protons have a thermal energy, explain if the reaction is possible, taking into account the principle of conservation of particles, spin, parity, etc.

d) Is the reaction possible at higher incident proton energies? Explain

Known values: $J^\pi(^7\text{Li}) = \frac{3}{2}^-$; $J^\pi(p) = \frac{1}{2}^+$.

8. In an approximation describing an electron in a polar medium, it is assumed that the interaction between its charge and its polarizable environment leads to the localization of the electron in a small spherical region. The electron thus sees a Coulomb potential for $r \geq R$ as shown. Give a rough estimate of U, the energy of the localized electron and its radius R. (It is not necessary to use the Schrödinger equation).



9. Using the data given below and the equilibrium between gravitational forces and pressure forces (hydrostatic equilibrium) answer the following questions:

a) Estimate the pressure at the center of the sun.

b) Assuming that the solar energy sources are thermonuclear reactions transmuting four protons into an α-particle estimate the potential lifetime of our sun. Neglect the energy loss due to neutrinos.

Mass	$M_\odot \simeq 2 \times 10^{33}$ g	$m_p \simeq 931$ MeV
Radius	$R_\odot \simeq 7 \times 10^{10}$ cm	$m_\alpha \simeq 3697$ MeV
Luminosity	$L_\odot \simeq 4 \times 10^{33}$ ergs/sec	$G \simeq 6.7 \times 10^{-8}$ dynes-cm^2-g^{-2}