CANADIAN ASSOCIATION OF PHYSICISTS

UNIVERSITY PRIZE EXAMINATION

Tuesday, January 17, 1978.

2:00 p.m. to 5:00 p.m.

Completed examination booklets should be sent by Department Chairmen to:

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Physics Department
Simon Fraser University
Burnaby, British Columbia
V5A 1S6

INSTRUCTIONS AND INFORMATION

1. Slide rules or pocket electronic calculators only are allowed.

2. ANSWER EACH QUESTION IN A SEPARATE BOOKLET with the question number and your name on the outer page of each booklet.

3. Answer as many questions as you can in whole or in part.

4. All the questions have the same weight. Your final mark will be the sum of the eight highest marks obtained.

5. $$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2}$$

6. Ratio of charge to rest-mass for the electron is

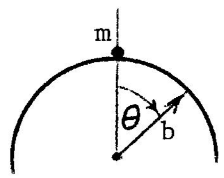
   $\frac{e}{m_o} = 1.76 \times 10^{11}$ coulombs/kg $= 5.27 \times 10^{17}$ stat coul/gm (e.s.u./gm)

   and $m_o = 9.11 \times 10^{-31}$ kg.

   Rest energy of the electron $= 5.11 \times 10^5$ eV.

7. Planck's constant $= 6.63 \times 10^{-34}$ joule-sec.

1. a) Estimate the pressure at the centre of the earth. You may disregard the rotation of the earth. State any assumptions you make.

   b) 

   A particle of mass m is placed at the uppermost point on the surface of a frictionless sphere of radius b. The particle is given a very small horizontal velocity. Calculate the angle θ at which the particle leaves the surface of the sphere.

2. An electron is moving in a plane circular orbit in a uniform magnetic field of 0.1 W/m^2 (10^3 gauss). The radius of the orbit is 1.12 cm. Calculate:

   1) the velocity of the electron, and

   2) the energy of the electron in eV.

   Determine whether electrons having this energy could be used in an electron microscope having a resolving power of 10^{-10} meter.

3. A simple classical model of an atom consists of a point positive charge (the nucleus) surrounded by a spherically symmetric cloud of negative charge in which the density is constant out to the atomic radius R_o and zero at larger radii. If the atom is placed in an electric field $\vec{E}$ there will be an induced dipole moment.

   a) Assuming that there is no distortion of the charge cloud by the electric field, derive an expression for the atomic polarizability of the atom in terms of the atomic radius R_o.

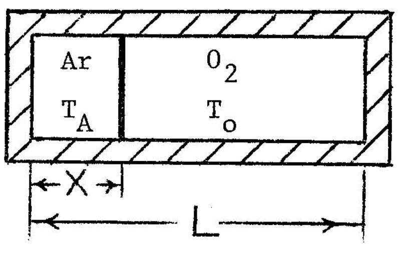
b) The dielectric constant for a gas of helium atoms (density $N = 2.7 \times 10^{19}/\text{cm}^3$) is known from experiment to be $\varepsilon = 1.00007$. Use the results of Part a) to estimate the atomic radius R_o for Helium.

4. The deuteron ground state is assumed to be an s-state (i.e. zero orbital angular momentum) and the binding energy is E_B. Consider a model in which the nuclear potential between two nucleons a distance r apart is taken to be

$$V(r) = -V_o \qquad 0 < r < b$$
$$= 0 \qquad r > b$$

Find the radial wave function and the energy eigenvalue relation for the deuteron ground state.

5. A thermally insulated, cylindrical container of length L is separated into two compartments by a thin partition initially thermally insulating and clamped at a distance X from the left end as shown below. The left compartment contains one mole of argon gas and the right compartment contains one mole of oxygen gas. The gases initially are at different temperatures and different pressures. (Neglect the heat capacity of the partition.)



a) Assume that the partition becomes thermally conducting and that it is unclamped so that it is free to move. Show that (even for non-ideal gases) the Helmholtz free energy for the total system ($F = E - TS$) is a minimum at the new equilibrium position, i.e. prove

$$\left(\frac{\partial F}{\partial X}\right)_{X=X_o} = 0, \quad X_o = \text{final equilibrium position.}$$

b) Assume the insulated partition is initially located at $X = \frac{1}{2} L$ where T_A = the initial temperature of the argon and T_o = the initial temperature of the O_2. The partition is now completely removed and the system is allowed to come to equilibrium. Find the final temperature of the mixture and the change in total entropy of the system. Assume the gases are ideal for this part.

c) Assume that both compartments have the same volume and each contains one mole of argon at the same temperature. What happens to the entropy of the total system when the partition is removed.

6\. a) State the third law of thermodynamics.

b)

______________ $\varepsilon_o + 100\ k\theta$

______________ $\varepsilon_o + k\theta$

______________ ε_o

The adjacent figure (not drawn to scale) represents the lowest few energy levels of a system which obeys Maxwell-Boltzmann statistics. The energy levels are non-degenerate. k is Boltzmann's constant.

When the temperature T is less than 20θ only the lowest two levels are important for statistical calculations. Show for $T < 20\theta$ that the specific heat at constant volume is given by

$$c_v = k\, y^2 e^{-y}/(1 + e^{-y})^2$$

where $y = \theta/T$.

At what temperature does this specific heat have its maximum value? Give your answer to two significant figures.

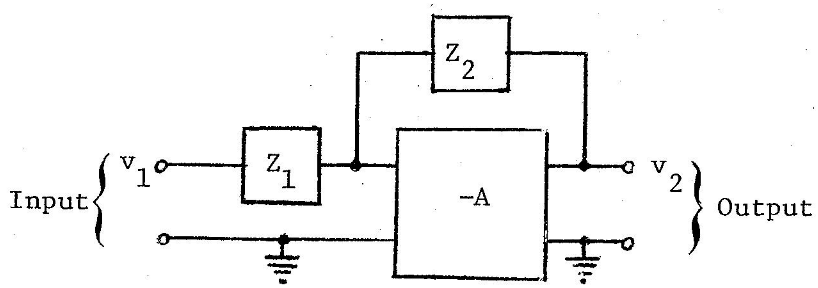
What is the significance of the above result for c_v from the point of view of the experimental verification of the third law of thermodynamics?

7. A bat is flying at right angles to a plane interface beyond which the air temperature rises by 10°C. The bat emits a forward beam of high frequency sound of 0.1 W power and of semi-angle 5° and its threshold of detection for this sound is 10^{-7} W m^{-2}.
Estimate at what distance away from the boundary the bat will become aware of the hot air region.

8. a) A microwave detector tuned to receive microwaves of wavelength 23 cm is located at the shore of a large lake 55 cm above the water level. A quasar, generating microwaves of constant intensity, slowly rises above the horizon on the side of the lake opposite to the detector. As the quasar rises the detector indicates successive maxima and minima of signal intensity. At what angle θ above the horizon is the quasar when the first maximum is received?

b) Blood cells are approximately uniform circular platelets 1.0×10^{-5} meters in diameter and can be made to lie flat against a glass surface. Describe the diffraction pattern obtained from a random array of such cells on a glass plate when a parallel light beam of wavelength 5×10^{-7} meters is incident normally on the plate and a lens of focal length 60 cm is used to focus the diffraction pattern. Describe the change which occurs in the diffraction pattern when the plate is rotated about an axis in its own plane.

9. An amplifier with amplification, −A, and infinite input impedance is connected to an impedance network as in the diagram.



a) Derive the exact relationship between the sinusoidal voltages, v_1 and v_2, and also the approximate relationship when A >> 1.

b) Suppose the input signal v_1 is a square wave of frequency 1000 Hz. How would you choose Z_1 and Z_2 so as to obtain a triangular wave output bearing in mind that for a real amplifier v_2 cannot exceed a certain saturation value $v_2 = v_{SAT}$? You may assume A >> 1 in the absence of saturation?