CANADIAN ASSOCIATION OF PHYSICISTS

UNIVERSITY PRIZE EXAMINATION

Monday, March 7, 1977

2 p.m. to 5 p.m.

Completed examination booklets should be sent by

Department Chairmen to:

Dr. R. G. Summers-Gill
Tandem Accelerator Laboratory
McMaster University
Hamilton, Ontario
L8S 4K1

INSTRUCTIONS:

1. Slide rules or pocket electronic calculators only are allowed.

2. ANSWER EACH QUESTION IN A SEPARATE BOOKLET, with the question number and your name on the outer page of each question.

3. Answer as many questions as you can, in whole or in part. Marks are indicated for each question.

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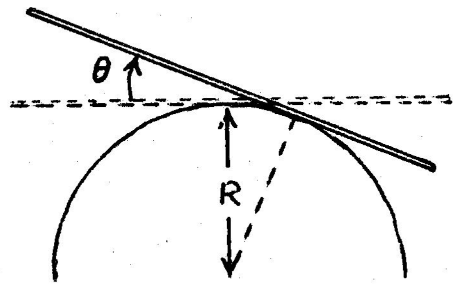
1. A uniform rigid bar (mass M, length 2a, negligible thickness) is balanced on a fixed horizontal cylinder (radius R) as shown in the figure. If the bar is tilted by a small angle θ from its horizontal equilibrium position, it will oscillate. Find the frequency of these small oscillations, assuming the bar does not slip.



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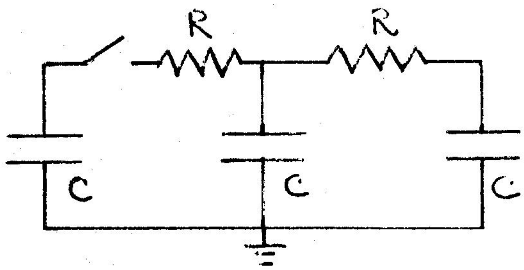
2. Initially the capacitor on the left has a charge Q and the other two capacitors are uncharged. At t=0 the switch is closed. Find the subsequent currents in each of the resistors and hence show that, when equilibrium has been established, the total heat produced in the lefthand resistor is seven times that produced in the righthand resistor.



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3. a) Using the Schrödinger equation in one dimension, show that for a particle of mass m in the potential well

$$V = \infty \text{ for } x < 0 \text{ and } x > a$$

$$V = 0 \text{ for } 0 \leq x \leq a,$$

the energy eigenvalues are

$$E_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{a}\right)^2 \qquad n = 1, 2, 3, \ldots \; .$$

b) Now consider a perfectly elastic ball bouncing perpendicularly back and forth between two vertical plane parallel walls separated by a distance a. Using classical mechanics, compute the time rate of change in energy of the ball as the walls are very slowly moved closer together at a relative speed u.

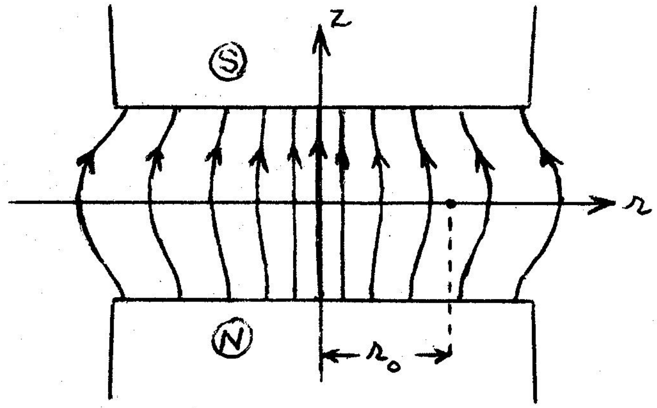
c) Show that the rate of change in problem (b) is consistent with what one has quantum-mechanically if the ball's quantum number n does not change.

8. In a conventional cyclotron, charged particles move in an evacuated chamber and are acted upon by a time-independent magnetic field B. A properly phased r.f. electric field accelerates the particles and as their energy increases they move on paths of successively larger radius until the orbit reaches the outside limits of the magnetic field region.

a) How does the radius of the orbit depend on the magnetic field strength, the properties of the ions being accelerated and their energy?

b) Show that the time taken for a complete revolution is independent of
25 the orbit radius and hence find the cycloton resonance frequency for protons.

The resonance condition found in (b) makes acceleration possible but does not guarantee that the beam will have a useful intensity. In addition, it is necessary that there be focussing so that particles which are above or below the median plane, z=0, in the magnet gap will be returned to it (vertical focussing) and particles which are inside or outside the "equilibrium" orbit, $r=r_o$, for their energy will be returned to the proper radius (radial focussing). To achieve this while retaining azimuthal symmetry, the magnetic guiding field must be non-uniform - falling off at increasing radius. In cylindrical co-ordinates, assume



$$B_z = B_o \{1-n\,(\frac{r-r_o}{r_o})\}$$

to first order in the small quantity $r-r_o \equiv \rho$, where B_o is a constant, the field necessary to achieve an orbit of radius r_o, and n is a number whose value is to be discovered.

c) Deduce the dependence of B_r on z to satisfy Maxwell's equations in the magnet gap.

d) Now set up the equations of motion for a particle of mass m and charge q moving in such a field. Subject to the constraint $r\,\frac{d\theta}{dt}$ = constant and for $|r-r_o|$ and $|z|$ small, show that

$$\left.\begin{aligned} \frac{d^2\rho}{d\theta^2} + (1-n)\,\rho \gtrsim 0 \\ \frac{d^2z}{d\theta^2} + nz \gtrsim 0. \end{aligned}\right\} \quad \text{to first order in } \rho \text{ and } z.$$

e) For what range of values of n will the solutions for ρ and z be simultaneously bounded?

9. A laser beam of 1 cm diameter has an output power of 100 megawatts and a
15 beam divergence of 1×10^{-3} radian. For a certain application, it is required to produce an electric field of 10^8 V/cm, which can be achieved by focussing the beam. Find the focal length of a lens which, when used to focus the beam, will produce at the focal plane of the lens the required field of 10^8 V/cm. (The permittivity constant $\varepsilon_o = 8.89 \times 10^{-12}$ in MKSA units.)