CANADIAN ASSOCIATION OF PHYSICISTS

UNIVERSITY PRIZE EXAMINATION

Tuesday, February 3, 1976

2 p.m. to 5 p.m.

Completed examination booklets should be sent by Department Chairmen to:

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Dalhousie University
Halifax, Nova Scotia
B3H 3J5

INSTRUCTIONS:

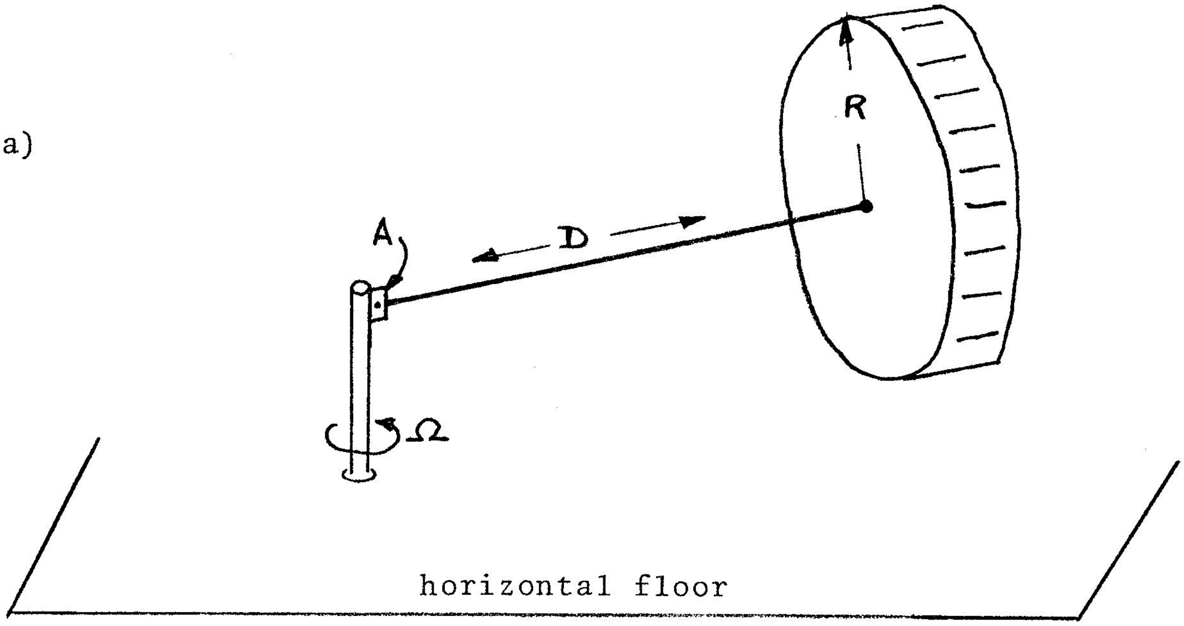
1. Slide rules or pocket electronic calculators only are allowed.

2. ANSWER EACH QUESTION IN A SEPARATE BOOKLET, with the question number and your name on the outer page of each question.

3. Answer as many questions as you can, in whole or in part. Marks are indicated for each question.

MARKS

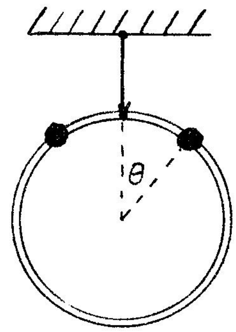
1. a)



(10) The above figure is a schematic drawing of a mechanism frequently used in mills (for crushing grain) which was known even in Roman times. The horizontal shaft (length D) of the millstone (mass M, radius R) is driven by a vertical shaft. The jointed connection at A enables the millstone to be tilted up or down in the vertical plane. The millstone rolls without slipping on the mill floor.

Explain why the total force exerted by the millstone on the mill floor is greater when the mill is in rotation than when it is at rest, and obtain an expression for the increase in force when the driving shaft is rotated with constant angular velocity Ω.

b) A ring of mass M hangs from a thread, and two beads of mass m slide on it without friction (see right). The beads are released simultaneously from the top of the ring and slide down opposite sides.



(10) Show that the ring will start to rise if $m > 3M/2$ and find the angle at which this occurs.

MARKS

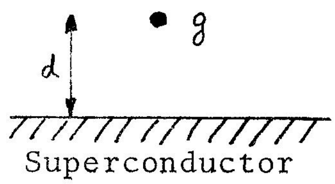
2. a) A meter bar of mass m is supported in the earth's gravitational field by a frictionless pivot at the bar's centre of mass. Find the period of small amplitude oscillations in a vertical
(15) plane about the position of stable equilibrium. Assume that the mass distribution of the earth is spherically symmetric.

b) A long string under uniform tension has a mass per unit length of 2 g cm^{-1}. Somewhere along its length
(10) a 5 g point mass is attached. A wave of wavelength 10 cm is incident on the point mass from one side. What fraction of the incident energy is reflected by the point mass?

3. A particle of rest mass m moves back and forth with velocity $\pm v$ along the x-axis, between two perfectly
(15) reflecting walls located at $x = 0$ and $x = L$. The time-averaged momentum of the particle is zero, and the time averaged energy is $\langle E \rangle = mc^2[1 - v^2/c^2]^{-\frac{1}{2}}$.

Find the time averaged momentum and energy of the particle in a coordinate system in which the walls have velocity u in the x-direction. The velocities v and u are both comparable to the velocity of light c.

4. a) A magnetic monopole g is located a distance d from a superconducting half-space. Find the force on
(5) g and the current density in the superconductor.



Note: The magnetic field generated by a magnetic monopole g is $B = g/r^2$, and if placed in a magnetic field B, a magnetic monopole g feels a force gB.

b) A conducting sphere of radius R floats exactly half submerged in a very large reservoir of oil of
(5) dielectric constant ε. The sphere carries a total charge Q. Find the charge densities on the free and submerged portions of the sphere.

MARKS

4\. c) (i) Write down the non-relativistic equations of motion for a particle of charge e and mass m, moving in a uniform horizontal magnetic field B and a vertical (downward) gravitational field g.

(ii) Integrate the equations of motion to obtain the coordinates of the particle as functions of time, assuming that the particle is at the origin and

(15) has zero velocity at $t = 0$. In particular, show that the average motion is horizontal.

(iii) Ordinary matter, say a piece of metal, is composed of charged particles, which as you have just seen move horizontally in a horizontal B and a vertical g. Why then is it that ordinary matter falls in a gravitational field in the presence of a horizontal B?

5\. a) Three identical bodies of constant thermal capacity are at temperatures 300, 300, and 100 K. If no work

(15) or heat is supplied from outside, what is the highest temperature to which any one of the bodies can be raised by operation of heat engines?

(5) b) Estimate the temperature of a black leather seat of a convertible which reaches a steady state in the hot summer sun. Neglect heat transfer by conduction and convection.

The sun has an effective black body temperature of 6000 K, and subtends an angle of $\frac{1}{2}^0$ viewed from the earth.

c) If the anharmonic force, f, between two atoms is given by $f = -\alpha\xi + \beta\xi^2$, where ξ is the instantaneous displace-

(5) ment from the equilibrium distance (a) between the atoms, show that the high temperature thermal expansion coefficient is given by

$$\frac{\beta k}{a\alpha^2} ,$$

where k is the Boltzmann constant.

MARKS

(10) 6. a) Given that the volume flow rate $Q = \Delta v/\Delta t$ of liquid through a long cylindrical pipe (length = ℓ, radius = r) depends only on ℓ, r, ΔP (the pressure drop between the ends), and η (the viscosity, with units $N\text{-}s/m^2$), determine how Q depends on r.

b) The phase velocity v of surface waves of wavelength λ on water of depth d is given by

$$v^2 = \frac{g\lambda}{2\pi} \tanh \frac{2\pi d}{\lambda}$$

where g is the acceleration due to gravity. Explain the following observations:

(i) Waves arrive parallel to a gently sloping beach.

(15) (ii) The surface of the sea is rougher near headlands than in nearby bays.

(iii) A ship at sea barely feels a tsunami ("tidal wave") which subsequently demolishes a coastal village.

(iv) The hull speed of a boat is given approximately by $1.5\sqrt{\ell}$ m/s, where ℓ is the waterline length in meters.

(v) The angle between the "arms" of the wake of a boat travelling in deep water is 39^0, independent of the speed of the boat.

(10) 7. a) In Young's experiment with a double slit, suppose the two slits are very narrow and long, and spaced ½ mm apart. The intensity pattern is observed on a plane surface located 2 meters from the slits.

Describe, as quantitatively as you can, the intensity distribution on the observing plane, including the ratio of maximum to minimum intensity over one complete fringe spacing, in the following cases:

(i) The slits are illuminated by a plane wave (λ = 630 nm) of plane polarized light, polarized perpendicular to the slit length and incident normally to the plane of the slits.

MARKS

7\. a) (ii) As in (i), except that at one slit an optical retardation of $5/2\ \lambda$ is introduced.

(iii) As in (i), except that at one slit the amplitude is reduced by a factor of 2.

(iv) As in (i), except that at one slit the intensity is reduced by a factor of 2.

(v) As in (i), except that at one slit the plane of polarization is rotated by 45^0.

b) Consider the superposition of two plane monochromatic light waves of the same wavelength, going in the same direction.

W1 is left hand elliptically polarized, with the ratio of major to minor axes 3:1.

(10) W2 is right circularly polarized with an amplitude equal to that of the minor axis of the ellipse.

By shifting the phase of W2 relative to that of W1:

(i) Is it possible to make the sum linearly polarized? If so, what is its direction of polarization relative to W1, and what is the amplitude?

(ii) Is it possible to make the sum circularly polarized? If so, what is the handedness, and what is the amplitude?

8\. a) A particle of mass m moves in the one-dimensional potential well

$V(x) = \infty \qquad |x| > a$

(10) $V(x) = 0 \qquad b \leq x \leq a \quad \text{and} \quad -a \leq x \leq -b$

$V(x) = -V_0 \qquad |x| < b$

Obtain an implicit equation which relates the quantized positive energy levels to the parameters of the well.

MARKS

8. b) Consider the hamiltonian $H = H_0 + H_1$, where H_0 has a lowest eigenvalue E_0, and H_1 has only non-negative eigenvalues.

(15) (i) Show that the lowest eigenvalue of H is larger than E_0.

(ii) Show that for a given potential well, the energy of the lowest p-state lies above that of the lowest s-state.