CANADIAN ASSOCIATION OF PHYSICISTS

UNIVERSITY PRIZE EXAMINATION

Tuesday, February 4, 1975

2 p.m. to 5 p.m.

Completed examination booklets should be sent by

Department Chairman to:

Dr. W. R. Conkie
Department of Physics
Queen's University
Kingston, Ontario
K7L 3N6

INSTRUCTIONS:

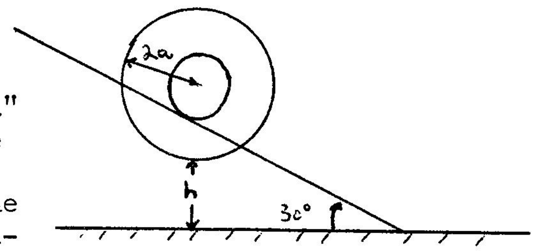
1. Slide rules only are allowed.

2. ANSWER EACH QUESTION IN A SEPARATE BOOKLET, with the question number and your name on the outer page of each question.

3. Answer as many questions as you can, in whole or in part. Marks are indicated for each question.

Marks

1. The axle of a flywheel rests on two straight, parallel rods, inclined at an angle of 30^o to the horizontal, so the flywheel is free to rotate between them, as shown. The axle has radius "a" the cylindrical exterior surface of the flywheel has radius 2a, its mass is m, and its moment of inertia, including the axle, is $I = 2ma^2$. The flywheel is released in the position shown, with its lowest edge a height "h" above the surface of the table. It rolls without slipping down the rods and then out onto the level table, where it eventually rolls without slipping at a constant speed v_f. (15)



Find v_f.

2.(a) A particle of mass m is connected to a particle of mass M by an elastic string of natural (unstretched) length L_o and spring constant (in tension only) k. The mass m is initially held fixed in position with the mass M suspended at rest beneath it on the string. If the system is now allowed to fall freely, at what time will the two masses collide? (15)

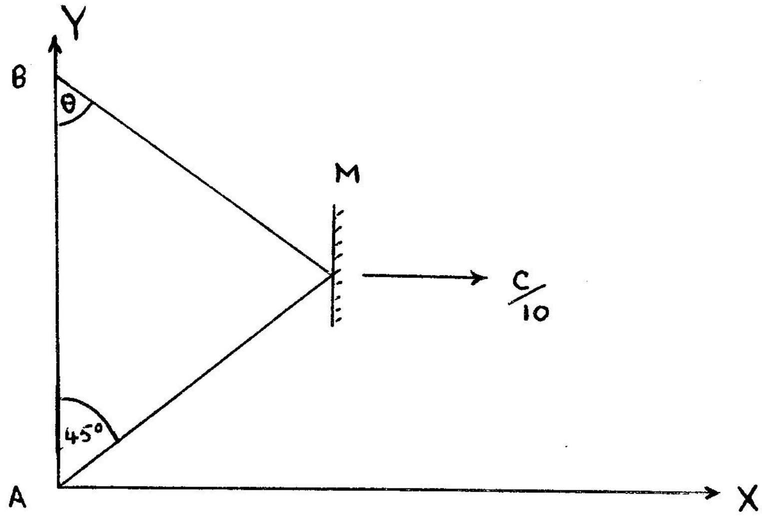
(b) A particle is constrained to move along the x-axis of a cartesian co-ordinate system and an identical particle is constrained to move along the y-axis. Show that if the particles are initially at rest and attract each other according to any law which depends only on the distance between them, then they will reach the origin simultaneously. (10)

3.(a) Two proton beams, each having a kinetic energy of 30 GeV, collide head on. If an equivalent collision is to be obtained by having a single beam collide with stationary protons, what must be the kinetic energy of the protons in this beam? (10)

Assume that the rest energy of a proton can be taken to be 1.0 GeV.

Marks

3.(b)



(10)

Light of wavelength 5000A emitted at 45° to the y axis by a laser at A, is reflected by the moving plane mirror M and observed at B.

(a) What is the angle θ? (It is <u>not</u> 45°!)

(b) What is the wavelength of the light observed at B?

4. The plane z = o separates two regions of space. In region 1 with z < o, the potential energy of a particle is constant with value V_1, in region 2 with z > o, the potential energy is also constant with value V_2. A quantum mechanical particle of mass m, wave vector (K_x, K_y, K_z) and energy E where $V_1<V_2<E$ is incident on the boundary from region 1. Draw an analogy between this situation and the situation in classical optics of light incident on a boundary separating two media with different refractive indices μ_1, μ_2 where

(20)

$$\frac{\mu_1}{\mu_2} = \sqrt{\frac{E - V_1}{E - V_2}} \quad .$$

Calculate the quantum mechanical transmission and reflection coefficients.

Show that the analogy includes the case of total internal reflection and that this occurs for

$$E < \frac{\hbar^2}{2m}(K_x^2 + K_y^2) + V_2 \quad .$$

Marks

5.(a) A thermally insulated vertical cylindrical vessel of cross-sectional area A containing a perfect gas is sealed near the top by a frictionless piston of mass m, which is free to move up and down. At equilibrium, the piston's weight is supported by a column of gas of height h inside the cylinder. If the piston is set to oscillate about its equilibrium position, show that the frequency of oscillation is given by

(10)

$$\nu = \frac{1}{2\pi} \sqrt{\frac{\gamma\,(p_o A + mg)}{mh}}$$

where p_o is the atmospheric pressure outside the cylinder and γ the ratio of specific heats of the gas. State whatever assumptions one must make to arrive at the result.

(b) Two weights are hung on the ends of a wire which passes over a block of ice. The wire is observed to pass gradually through the block of ice in a process called "regelation". Also it is observed experimentally that heat must be added to convert ice to water. Use these facts to deduce that ice should float on water.

(5)

6. The Planck black-body law gives the power radiated from a black surface per unit area and per unit wavelength interval to be $W_\lambda = c_1\lambda^{-5}\,(e^{c_2/\lambda T} - 1)^{-1}$ where $c_1 = 2\pi hc^2 = 3.74 \times 10^{-16}\,Wm^2$ and $c_2 = hc/k = 1.439 \times 10^{-2}\,mK$. A He-Ne gas laser operating on the 632.8 nm red line has an effective linewidth of 1.0×10^{-6} nm, a power output of 4.8 mW and an output beam with a cross-sectional area of 8.0 mm^2. One end mirror is almost perfectly reflecting and only 0.99% of the flux of 632.8 nm light incident on the exit end window escapes.

(10) (a) Calculate the effective surface temperature of the black body which appears to be as bright as the laser over its effective linewidth at 632.8 nm.

(5) (b) Show that the ratio of the Einstein coefficients for spontaneous and stimulated emission respectively is given by $A/B = 8\pi h\lambda^{-3}$.

(5) (c) If the effective kinetic temperature of the plasma is given by $T = 0.5\,c_2/632.8$ nm, estimate the number of induced emissions that take place for every spontaneous emission over the wavelength range of the laser in that part of the active medium through which the laser light is passing.

Marks

7. Consider two long parallel straight wires separated by a distance r and bearing equal and opposite uniform charge distributions. In the rest frame of the wires an observer S notes that there is no current flowing and that the linear charge density is λ in magnitude (i.e. λ coulombs per metre length). Another observer
(20) S' sees observer S and the wires moving parallel to their length with velocity v.

(a) Find the linear charge density λ' and the current i' in the wires (if any) as measured by observer S'.

(b) Calculate the electric and magnetic fields at a point midway between the wires as observed by S and S' respectively and compare.

(c) Calculate the force per unit length between the wires as observed by S and S' respectively and compare.

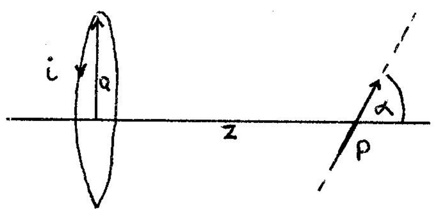
8. Two infinite, plane, parallel plates of metal are separated by a distance d. The space between the plates is filled with two conducting media, the interface between the media being a plane parallel to the plates. The two media have conductivity,
(10) permittivity and thickness σ_1, ε_1, α and σ_2, ε_2, $(d-\alpha)$ respectively. The metal plates are maintained at potentials V_1, V_2 respectively. In the steady state,

(i) What is the potential of the interface between the media?

(ii) What is the surface density of charge on the interface?

(b)

(10)



A small magnet (with magnetic moment m and moment of inertia I) is placed at a distance z on the axis of a loop of wire of radius a, carrying a current i. Suppose that the centre of the magnet is fixed at P. Calculate the period of its small oscillation.

Marks

9.(a) (15) An electron is contained inside a sphere of radius R. What is the pressure exerted on the surface of the sphere if the electron is in the lowest S state?

(b) (10) Consider the one-dimensional Schrödinger equation with

$$V(x) = \begin{cases} +\infty & x < 0 \\ \frac{1}{2} m\omega^2 x^2 & x > 0 \end{cases}$$

What are the energy eigen-values?